10-3 Properties of Logarithms

Objective: Simplify and evaluate expressions using the properties of logarithms.
Solve logarithmic equations using the properties of logs.

I. Properties

1. **Product**: $\log_b mn = \log_b m + \log_b n$

2. **Quotient**: $\log_b (m/n) = \log_b m - \log_b n$

3. **Power:** $\log_b m^p = p \times \log_b m$

EX 1. Use $\log_5 2 = .4307$ to approximate the value of $\log_5 250$.

$\log_5 5^x = x$

$\log_5(250) = \log_5(2 \cdot 125) = \log_5 2 + \log_5 125$

$= .4307 + \log_5 5^3 = .4307 + 3 =$

$= 3.4307$

EX 2. Use $\log_6 8 = 1.106$ and $\log_6 32 = 1.9343$ to approximate the value of $\log_6 4$.

$\log_6 6^x = x$

$\log_6 4 = \log_6 \frac{32}{8} = \log_6 32 - \log_6 8$

$= 1.9343 - 1.106 = .8283$

EX 3. Given $\log_5 6 = 1.1133$, approximate the value of $\log_5 216$.

$\log_5 5^x = x$

$\log_5 216 = \log_5 6^3 =$

$= 3 \log_5 6$

$= 3(1.1133) = 3.3399$

II. **Solve**.

EX 4. $4\log_2 x - \log_2 5 = \log_2 125$

$\log_2 x^4 - \log_2 5 = \log_2 125$

$\log_2 \frac{x^4}{5} = \log_2 125$

$5 \cdot \frac{x^4}{5} = 125 \cdot 5$

$\sqrt[4]{x^4} = \sqrt[4]{625}$

$x = \pm 5$

$x = 5$ only

since you can't take the log of a (-) #.

EX 5. $\log_8 x + \log_8 (x-12) = 2$

$\log_8 (x(x-12)) = 2$

$8^2 = x(x-12)$

$64 = x^2 - 12x$

$0 = x^2 - 12x - 64$

$0 = (x-16)(x+4)$

$x = 16 \quad x = -4$

EX 6. $3\log_5 x - \log_5 4 = \log_5 16$

$\log_5 x^3 - \log_5 4 = \log_5 16$

$\log_5 \frac{x^3}{4} = \log_5 16$

$4 \cdot \frac{x^3}{4} = 16 \cdot 4$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

EX 7. $\log_4 x + \log_4 (x-6) = 2$

$\log_4 (x \cdot (x-6)) = 2$

$4^2 = x(x-6)$

$16 = x^2 - 6x$

$0 = x^2 - 6x - 16$

$0 = (x-8)(x+2)$

$x = 8$ $\quad$ ~~$x = -2$~~