10-5 Base e and Natural Logarithms

Objective: Evaluate expressions involving the natural base and natural logs.
Solve exponential equations and inequalities using natural logs.

I. Intro- P.554 What is e?

If $y=\ln x$, then $e^y=x$.

Recall
$2^3=8$
$\log_2 8=3$

II. Evaluate- 4 decimal places.

Ex 1. $e^{.5} \approx$ 1.6487

EX 2. $e^{-8} \approx$ .0003
3.3546 E-4

EX 3. $\ln 3 \approx$ 1.0986

Ex 4. $\ln(1/4) \approx$ -1.3863

III. Write equivalent expressions.

EX 5. $e^x=23$ $\ln 23 = x$

Ex 6. $\ln x=1.2528$ $e^{1.2528} = x$

IV. Inverse Property of Base e and natural logs.

$e^{\ln x} = x$ and $\ln e^{x} = x$

EX 7. $e^{\ln 21} = 21$

Ex 8. $\ln e^{x^2 - 1} = x^2 - 1$

Recall

$2^{\log_2 5} = 5$

$\log_4 4^{90} = 90$

V. Solve (all prop. of logs apply to natural logs as well).

EX 9. $3e^{-2x} + 4 = 10$

$-4 \quad -4$

$\frac{3e^{-2x}}{3} = \frac{6}{3}$

$e^{-2x} = 2$

$\ln 2 = -2x$

$\frac{\ln 2}{-2} = \frac{-2x}{-2}$

$x = \frac{\ln(2)}{-2}$

$x = -.3466$

Review

$\log_b(xy) = \log_b x + \log_b y$

$\log_b \frac{x}{y} = \log_b x - \log_b y$

$\log_b x^n = n \cdot \log_b x$

EX 10. $\ln 3x = .5$

$e^{(.5)} = 3x$

$\frac{e^{(.5)}}{3} = \frac{3x}{3}$

$.5496 = x$

EX 11. $\ln(2x-3) < 2.5$

$2x-3 < e^{2.5}$

$2x < e^{2.5} + 3$

$x < \frac{(e^{2.5}+3)}{2}$

But $2x-3 > 0$

$2x > 3$

$x > \frac{3}{2}$

$1.5 < x < 7.5912$

Recall

$\log_2 x < 5$

$x < 2^5$

$0 < x < 32$

EX 12. $700 is deposited into an account paying 6% annual interest compounded continuously.

Need

$A = Pe^{rt}$

A=amount after t years, P=principal. (principle)

r=rate, t=years. (decimal)

A. What's the balance after 8 years?

$A = pe^{rt}$

$A = 700e^{(.06*8)} = \$1131.25$

B. How long will it take to grow to a least $2000?

$A = pe^{rt}$

$\frac{2000}{700} = \frac{700e^{(.06\cdot t)}}{700}$

$\frac{20}{7} = e^{(.06t)}$

$\frac{\ln(\frac{20}{7})}{.06} = \frac{.06t}{.06}$

$17.5 = t$

After 17.5 yrs