10-6 Exponential Growth and Decay

Objective: Use logs to solve problems involving exponential decay and growth

I. Exponential decay of the form $\boxed{y = a(1 - r)^t}$

New amount — rate of decrease (decimal)

initial amt — time

Example 1: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for 90% of this caffeine to be eliminated from a person's body?

(looking for time

$130(.10) = 13$

$y = a(1-r)^t$

$\frac{13}{130} = \frac{\cancel{130}(1-.11)^t}{\cancel{130}}$

$.1 = .89^t$

$\log .1 = \log .89^t$

$\log .1 = t \cdot \log .89$

$\frac{\log(.1)}{\log(.89)} = t = \boxed{19.75 \text{ hrs}}$

I. Exponential Decay of the form $\boxed{y = ae^{-kt}}$

New amt

initial amt

Time

k=constant (-) decay

*Scientists prefer this one

Example 2: The half-life of Sodium-22 is 2.6 years. The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated.

a=can be any #

a) What is the value of k for Sodium-22?

$$y = ae^{kt}$$
$$\frac{2}{4} = \frac{4}{4}e^{(k \cdot 2.6)}$$
$$.5 = e^{2.6k}$$

$$\ln(.5) = 2.6k$$
$$\frac{\ln .5}{2.6} = \boxed{k = -.2666}$$

b) A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth?

"t"

$$y = ae^{kt}$$
$$10 = 100e^{(-.2666t)}$$
$$.1 = e^{(-.2666t)}$$

$$\ln .1 = -.2666t$$
$$\frac{\ln .1}{-.2666} = \boxed{t = 8.64 \text{ yrs}}$$

I. <u>Exponential Growth of the form</u> $y = \boxed{a(1+r)^t}$

New amt — initial amt — rate of increase — Time

Example 3: The population of a city of one million is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled?

"t"

$$y = a(1+r)^t$$

$$2{,}000{,}000 = 1{,}000{,}000(1+.03)^t$$

$$2 = (1.03)^t$$

$$\log 2 = \log(1.03)^t$$

$$\log 2 = t \log(1.03)$$

$$\frac{\log 2}{\log(1.03)} = t = \boxed{23.45 \text{ years}}$$

I. Exponential Growth of the form $\boxed{y = ae^{kt}}$

New amt — initial amt — k=constant — Time

Scientists prefer this one

Example 4: As of 2000, Nigeria had an estimated population of 127 million people and the United States had an estimated population of 278 million people. The populations of Nigeria and the United States can be modeled by $N(t) = 127e^{.026t}$ and $U(t) = 278e^{.009t}$, respectively. According to these models, when will Nigeria's population be more than the population of the United States?

$$127e^{.026t} > 278e^{.009t}$$

$$\ln(127e^{.026t}) > \ln(278e^{.009t})$$

$$\ln 127 + \ln e^{.026t} > \ln 278 + \ln e^{.009t}$$

$$\ln 127 + .026t > \ln 278 + .009t$$

$$\underbrace{.026t - .009t} > \ln 278 - \ln 127$$

$$.017t > \ln 278 - \ln 127$$

$$t > \frac{(\ln(278) - \ln(127))}{.017}$$

$$t > 46.08$$

After 46 yrs

or

in the year 2046