11-2 Arithmetic Series

"Sum"

Objective: Find sums of arithmetic series.
Use sigma notation.

Series: An indicated sum of the terms of a sequence.

$d = 4$

18, 22, 26, 30-Arithmetic sequence.

18+22+26+30-arithmetc series.

Sum of Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

n = how many terms adding; a_1 = 1st term; a_n = last term

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

d = common difference

EX 1. Find the sum of the first 20 even numbers, beginning with 2.

2, 4, 6, 8 ... 40

$n = 20$

$a_1 = 2$

$a_{20} = 40$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{20} = \frac{20}{2}(2 + 40) = 10(42)$

$= 420$

EX 2. Find the sum of the first 100 positive integers.

$1 + 2 + 3 + 4 \ldots + 100$

$n = 100$

$a_1 = 1$

$a_{100} = 100$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{100} = \frac{100}{2}(1 + 100)$

$= 50(101) = 5050$

EX 3. A radio station considered giving away ~~$4000~~ 31 days everyday in August, which would equal $124,000. But they decided to increase the amount given everyday, but give the same total. They want to increase $100 a day, so how much should they give away the first day?

We need a_1. $n=31$, $S_{31}=124{,}000, d=100$

$$S_n=\frac{n}{2}[2a_1+(n-1)d]$$

$$124{,}000=\frac{31}{2}[2a_1+(31-1)100]$$

$$\frac{124{,}000}{15.5}=\frac{15.5}{15.5}[2a_1+3000]$$

$$8000=2a_1+3000$$

$$5000=2a_1$$

$a_1=2{,}500$ on day 1

EX 4. Find the first 4 terms of the arithmetic sequence in which $a_1=14$, $a_n=29$, $S_n=129$.

14, 17, 20, 23 (+3, +3, +3)

We need "d".

Step 1

$$S_n=\frac{n}{2}(a_1+a_n)$$

$$129=\frac{n}{2}(14+29)$$

$$\frac{129}{43}=\frac{n}{2}\cdot\frac{43}{43}$$

$$3=\frac{n}{2}$$

$$6=n$$

Step 2

$$a_n=a_1+(n-1)d$$

$$29=14+(6-1)d$$

$$15=5d$$

$$3=d$$

Sigma Notation-shortens writing out series.

Summation Notation

upper limit

Σ formula

lower limit

Ex 5. 3+6+9+12+...+30.

$\sum_{1}^{10} 3n$

Need formula

$a_n = a_1 + (n-1)d$

$a_n = 3 + (n-1)3 = 3 + 3n - 3$

$= 3n$ — formula

EX 6. Evaluate $\sum_{k=3}^{10} (2k+1)$

$= (2\cdot3+1) + (2\cdot4+1) + (2\cdot5+1) + (2\cdot6+1)$
$+ (2\cdot7+1) + (2\cdot8+1) + (2\cdot9+1) + (2\cdot10+1)$

$= 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

$= 112$

EX 7. $\sum_{j=5}^{8} (3j-4)$

$3\cdot5-4 = 11 = a_1$

$3\cdot8-4 = 20 = a_4$

Short Cut

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_4 = \frac{4}{2}(11+20)$

$= 2(31) = 62$

Ex 8. 6 + 13 + 20 + 27+ ... + 97

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{14} = \frac{14}{2}(6+97) =$

$7(103) = 721$

We need "n"

$a_n = a_1 + (n-1)d$

$97 = 6 + (n-1)7$

$97 = 6 + 7n - 7$

$97 = 7n - 1$

$98 = 7n$

$14 = n$