

11-3 Geometric Sequences

$r = \frac{1}{2}$
 $64, 32, 16, 8, \dots$

$r = 2$
 $4, 8, 16, 32, 64, \dots$

Objective: Use geometric sequences and find geometric means.

A **geometric sequence** is a sequence in which every term after the first is found by multiplying the previous term by a constant r, called the common ratio.

EX 1. $324, 108, 36, 12, \underline{4}$.
 $r = \frac{1}{3}$
 $r = \frac{a_2}{a_1} = \frac{108}{324} = \frac{1}{3}$

EX 2. $8, 20, 50, 125, \underline{312.5}$
 $r = \frac{a_2}{a_1} = \frac{20}{8} = 2.5$
No Rounding

nth term of a geometric sequence.

$a_n = a_1(r)^{n-1}$
 $n = \text{any positive integer.}$
 $a_n = \text{nth term}$ $a_1 = \text{first term}$ $r = \text{common ratio}$

EX 3. Find the 6th term of the geometric sequence for which $a_1 = -3, r = -2$. $n =$

Way 1: $\frac{-3}{a_1}, \frac{6}{a_2}, \frac{-12}{a_3}, \frac{24}{a_4}, \frac{-48}{a_5}, \frac{96}{a_6}$
 $a_6 = 96$

Way 2: $a_n = a_1(r)^{n-1} \rightarrow a_6 = -3(-2)^{6-1} = 96$

EX 4. Write an equation for the nth term of the geometric sequence 5, 10, 20, 40, ...

"General"
 $a_n = a_1(r)^{n-1}$
 $a_1 = 5, r = 2$
 $a_n = 5(2)^{n-1}$

EX 5. Find the 7th term of the geometric sequence for which $a_3 = 96, r = 2$. $n = 7$

$a_n = a_1(r)^{n-1}$
 $a_7 = 24(2)^{7-1}$
 $a_7 = 1536$

$\frac{24}{a_1}, \frac{48}{a_2}, \frac{96}{a_3}$

Geometric Means: are the missing terms between 2 non-successive terms of a geometric sequence.

EX 6. Find the 3 geometric means between 2.25 and 576.

$$\underline{r=4} \quad \boxed{2.25, \quad \underline{\quad}, \quad \underline{36}, \quad \underline{144}, \quad 576}$$

We need "r"!

$$a_n = a_1(r)^{n-1}$$

$$\frac{576}{2.25} = \frac{2.25(r)^{5-1}}{2.25}$$

$$\sqrt[4]{256} = r^4$$

$$r = \pm 4$$

OR

$$r = -4 \quad \boxed{2.25, \quad \underline{\quad}, \quad \underline{36}, \quad \underline{144}, \quad 576}$$

Condensed Answer

$$2.25 \quad \boxed{\pm 9, \quad 36, \quad \pm 144} \quad 576$$