11-5 Infinite Geometric Series

Objective: Find the sum of an infinite geometric series.
Write repeating decimals as fractions.

I. Read P.599 and go over **

Infinite geometric series: goes on forever.

Partial Sum: S_n is called a partial sum for this series.

Convergent series: An infinite series that has a sum.

II. Sum of an infinite geometric series.

$\boxed{S = \frac{a_1}{1-r}}$ $\boxed{-1<r<1}$ S=sum

Terms at the end of a sequence approach 0, so there will be a sum if $-1<r<1$.

EX 1. (1/2)+(3/8)+(9/32)+... Find the sum, if it exists.

Check "r": $r = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{\cancel{8}_4} \cdot \frac{\cancel{2}}{1} = \frac{3}{4}$

? $-1 < \frac{3}{4} < 1$ yes!

$S = \frac{a_1}{1-r} = \frac{1/2}{1-\frac{3}{4}} = \frac{.5}{.25} = \boxed{2}$

EX 2. 3-(3/2)+(3/)-(3/8)+...

Check "r"

$r = \frac{-\frac{3}{2}}{\frac{3}{1}} = -\frac{\cancel{3}}{2} \cdot \frac{1}{\cancel{3}} = \frac{-1}{2}$

? $-1 < -\frac{1}{2} < 1$ yes!

$S = \frac{a_1}{1-r} = \frac{3}{1-{}^-\frac{1}{2}} = \frac{3}{1.5} = \boxed{2}$

EX 3. (-4/3)+4-12+36-108+...

Check "r" $r = \frac{4}{-4/3} = \cancel{4} \cdot \frac{-3}{\cancel{4}} = -3$?

$-1 < -3 < 1$

No! $\boxed{\text{No Sum}}$

III. Infinite Series in Sigma Notation

EX 4. Evaluate $\sum_{n=1}^{\infty} 24(-1/5)^{n-1}$

Check "r" $r = -\frac{1}{5}$

$S = \frac{a_1}{1-r} = \frac{24}{1-\frac{-1}{5}} = \frac{24}{1.2} = 20$

EX 5. $\sum_{n=1}^{\infty} 5(1/2)^{n-1}$

$r = \frac{1}{2}$ $S = \frac{a_1}{1-r} = \frac{5}{1-\frac{1}{2}} = \frac{5}{.5} = 10$

IV. Write a repeating decimal as a fraction.

EX 6. .3939...

$.39 + .0039 + .000039 + \ldots$

$r = \frac{.0039}{.39}$

$= .01$

$\frac{a_1}{1-r} = \frac{.39}{1-.01} = \frac{.39^{\times 100}}{.99_{\times 100}} = \frac{39}{99}$

$= \boxed{\frac{13}{33}}$

EX 7. $.\cancel{25}25\ldots$ $.524524\ldots$

$= .524 + .000524 + .000000524 + \ldots$

$r = \frac{.000524}{.524}$

$r = .001$

$\frac{a_1}{1-r} = \frac{.524}{1-.001} = \frac{.524}{.999} = \frac{524}{999}$

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