

11-6 Recursion and Special Sequences

**Objective:** Recognize and use Special Sequences.  
Iterate functions.

**Fibonacci Sequence:** Each term in the sequence is a sum of the two previous terms...found many places in nature.

1, 1, 2, 3, 5, 8, 13, ... 21

**Recursive Formula:** Each term is formulated from one or more previous terms...must be given value(s) of first term(s) so you can start sequence.

EX 1. Find the first 5 terms of the sequence if  $a_1=4$ ,  $a_{n+1}=3a_n-2$ ,  $n \geq 1$ .

4, 10, 28, 82, 244  
 $3 \cdot 4 - 2$     $3 \cdot 10 - 2$     $3 \cdot 28 - 2$     $3 \cdot 82 - 2$

EX 2. Find the first 4 terms of the sequence if  $a_1=5$ ,  $a_{n+1}=2a_n+7$ ,  $n \geq 1$ .

5, 17, 41, 89  
 $2 \cdot 5 + 7$     $2 \cdot 17 + 7$     $2 \cdot 41 + 7$

EX 3. Mr. Yazaki discovered that there were 225 dandelions in his garden on the first Saturday of Spring. He had time to pull out 100, but by the next Saturday, there were twice as many as he had left. Each Saturday in spring, he removed 100 dandelions, only to find that the number of remaining dandelions had doubled by the following Saturday.

a. Write a recursive formula for the number of dandelions Mr. Yazaki finds in his garden each Saturday.

$d_n = \# \text{ of dandelions at the beginning of the } n^{\text{th}} \text{ Saturday!}$

$$d_{n+1} = 2(d_n - 100)$$

b. Find the number of dandelions Mr. Yazaki would find on the fifth Saturday.

$$d_1 = 225$$

$$d_2 = 2(225 - 100) = 250$$

$$d_3 = 2(250 - 100) = 300$$

$$d_4 = 2(300 - 100) = 400$$

$$d_5 = 2(400 - 100) = 600$$

dandelions



EX 4. Dr. Elliot is growing cells in lab dishes. She starts with 108 cells Monday morning and then removes 20 of these for her experiment. By Tuesday the remaining cells have multiplied by 1.5. She again removes 20. This pattern repeats each day in the week.

a. Write a recursive formula for the number of cells Dr. Elliot finds each day before she removes any.

$$C_n = \text{\# of cells beginning each day}$$

$$C_{n+1} = 1.5(C_n - 20)$$

b. Find the number of cells she will find on Friday morning.

$$M: 108$$

$$T: 1.5(108 - 20) = 132$$

$$W: 1.5(132 - 20) = 168$$

$$Th: 1.5(168 - 20) = 222$$

$$Fri: 1.5(222 - 20) = \boxed{303 \text{ cells}}$$

**Iteration:** on top of page 608

EX 6. Find the first 3 iterates  $x_1, x_2, x_3$  of the function  $f(x) = 3x - 1$  for an initial value of  $x_0 = 5$ .

$$f(5) = 3 \cdot 5 - 1 = 14 = x_1$$

$$f(14) = 3 \cdot 14 - 1 = 41 = x_2$$

$$f(41) = 3 \cdot 41 - 1 = 122 = x_3$$