12-2 Permutations and Combinations

Permutations: When a group of objects are in a certain order and order **DOES** matter. (telephone numbers)

$$P(n, r) = \frac{n!}{(n-r)!}$$

Total — picking

*n total objects picking r at a time

Example 1: Eight people entered a pie contest. How many ways can blue, red, and white be awarded?

order matters

$$P(8,3) = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 336 \text{ ways}$$

Example 2: There are 10 finalists in a skating competition. How many ways can gold, silver, and bronze be awarded?

order matters

$$P(10,3) = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 720 \text{ ways}$$

Permutations with Repetitions: $\frac{n!}{p!q!}$ ← total # of letters; repeats (any letter that repeats)

Example 3: How many different (distinguishable) ways to arrange letters of Mississippi?

4 – i's
4 – s's
2 – p's

$$= \frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 34{,}650 \text{ ways}$$

Combinations: Order is **NOT** important. (committee members)

$C(n, r) = \frac{n!}{(n-r)!\,r!}$

"n" total objects choosing "r" at a time

Example 4: Twenty people are at a birthday party. Three people need to pick up the pizza. How many ways to choose the people?

order doesn't matter

$C(20,3) = \frac{20!}{17!\,3!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17!\,3 \cdot 2 \cdot 1} =$ 1,140 ways

Example 5: Six cards are drawn from a deck of cards. How many hands consist of two hearts and four spades?

order does not matter

$C(13,2) \cdot C(13,4) = 78 \cdot 715 =$ 55,770 ways

hearts · spade

Example 6: Seven students in a group and 2 students need to present their project. How many ways can the students be chosen?

order does not matter

$C(7,2) = \frac{7!}{5!\,2!} = \frac{7 \cdot 6 \cdot 5!}{5!\,2 \cdot 1} = \frac{42}{2} = 21$

Example 7: Five cards are drawn from a deck of cards. How many hands consist of 3 clubs and 2 diamonds?

Clubs · diamonds

$C(13,3) \cdot C(13,2)$

$286 \cdot 78 =$ 22,308

on calculator

13 | Math | PRB | nCr | 3 = 286