12-2 Permutations and Combinations

Permutations: When a group of objects are in a certain order and order **DOES** matter. (telephone numbers)

$${}_nP_r = P(n, r) = \frac{n!}{(n-r)!}$$

n = total

r = # you are picking

Example 1: Eight people entered a pie contest. How many ways can blue, red, and white be awarded?

order matters

$$P(8,3) = \frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 336 \text{ ways}$$

$5!$

or $8 \cdot 7 \cdot 6 = 336$ ways

Example 2: There are 10 finalists in a skating competition. How many ways can gold, silver, and bronze be awarded?

order matters

$$P(10,3) = \frac{10!}{(10-3)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 720 \text{ ways}$$

$7!$

or

$10 \cdot 9 \cdot 8 = 720$

Permutations with Repetitions:

$$\frac{n!}{p!\,q!\ldots}$$

$n!$ ← total # of letters

$p!\,q!\ldots$ ← letters that repeat

Example 3: How many different (distinguishable) ways to arrange letters of Mississippi?

Mississippi
iMssissppi

$$\frac{11!}{4!\,4!\,2!} = \frac{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1\cdot2\cdot1}$$

"i" "s" "p"

$= 34{,}650$ ways

Combinations: Order is **NOT** important. (committee members)

$C(n, r) = \frac{n!}{(n-r)!\, r!}$

total Choosing

Example 4: Twenty people are at a birthday party. Three people need to pick up the pizza. How many ways to choose the people?

Order does not matter

$C(20,3) = \frac{20!}{17!3!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17!\, 3 \cdot 2 \cdot 1} = 20 \cdot 19 \cdot 3 = 1{,}140$ ways

Example 5: Six cards are drawn from a deck of cards. How many hands consist of two hearts and four spades?

order does not matter

hearts · Spades = $C(13,2) \cdot C(13,4)$

*Calc $78 \cdot 715 = 55{,}770$ ways

Example 6: Seven students in a group and 2 students need to present their project. How many ways can the students be chosen?

order does matter

$C(7,2) = \frac{7!}{5!2!} = \frac{7 \cdot 6 \cdot 5!}{5!\, 2 \cdot 1} = 7 \cdot 3 = 21$

Example 7: Five cards are drawn from a deck of cards. How many hands consist of 3 clubs and 2 diamonds?

Order does not matter

= Clubs · diamonds

= $C(13,3) \cdot C(13,2)$

$286 \cdot 78$

22,308 ways

*Calculator

Math

- PRB
- #2 nPr
- #3 nCr