13-3 Trigonometric Functions of General Angles

Objective: Find values of trig functions for general angles.
      Use reference angles to find values of trig functions.

Trig functions, angle in standard position

$r = \sqrt{(x^2 + y^2)}$
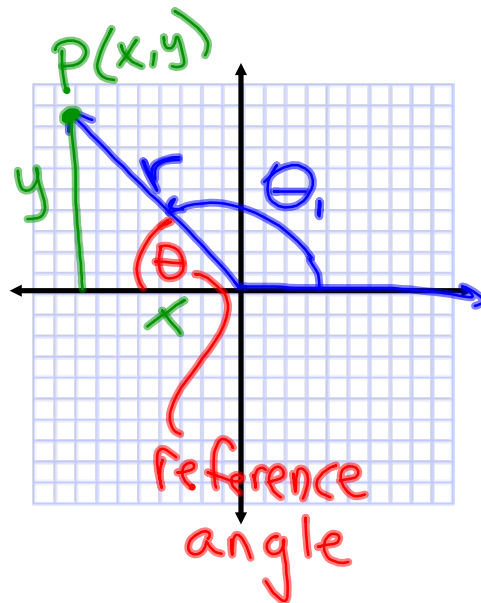
$\sin \theta = \dfrac{y}{r}$

$\cos \theta = \dfrac{x}{r}$

$\tan \theta = \dfrac{y}{x} \quad x \neq 0$

$\csc \theta = \dfrac{r}{y}$

$\sec \theta = \dfrac{r}{x} \quad y \neq 0$

$\cot \theta = \dfrac{x}{y} \quad x \neq 0$

$\dfrac{x}{y} \quad y \neq 0$

P(x,y)

y

r

θ₁

θ

x

reference
angle

Ex 1) Find the exact values of the 6 trig functions of θ if the terminal side of θ contains the point (5, -12).

$x \quad y$

$\sin\theta = \frac{y}{r} = \frac{-12}{13}$ $\csc\theta = \frac{13}{-12}$
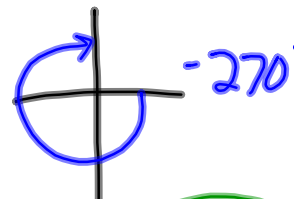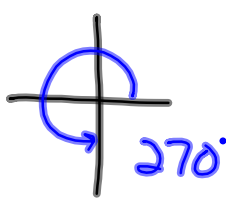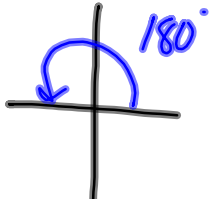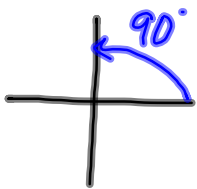
$r = \sqrt{5^2 + (-12)^2}$

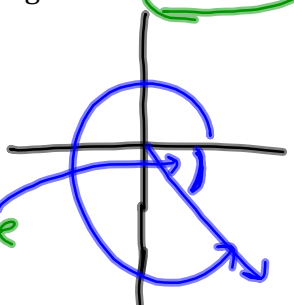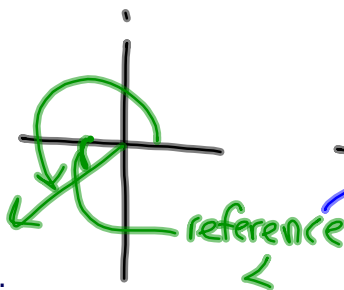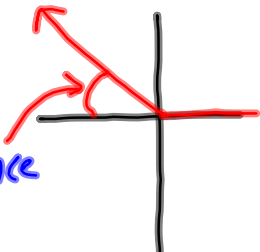$\cos\theta = \frac{x}{r} = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$
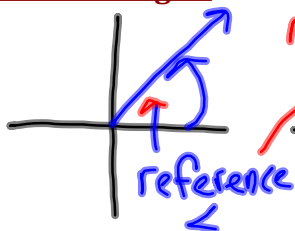
$r = 13$

$\tan\theta = \frac{y}{x} = \frac{-12}{5}$ $\cot\theta = \frac{5}{-12}$

Quandrantal Angles—have terminal side on the x- or y-axis.

90°   180°   270°   -270°

Reference Angles—acute angle formed by the terminal side of the angle and the x-axis.
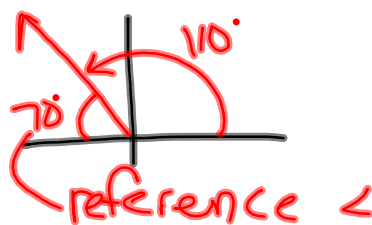
reference   reference
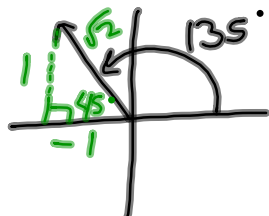
Ex 2) Sketch each angel and reference angle.

A) 330   B)-5∏   C) 110
                  6

330°   30°   30°   -150°   110°   70°

reference   reference
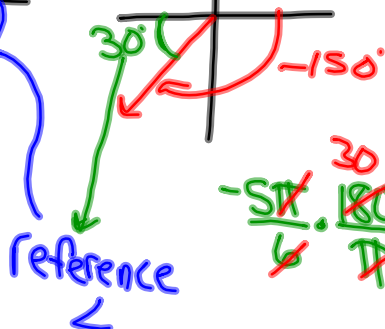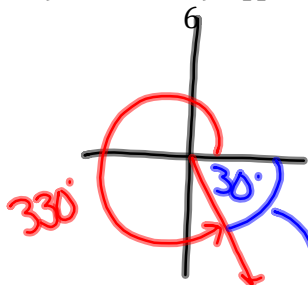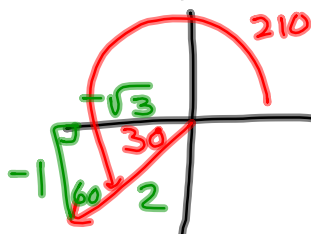
$\frac{-5\pi}{6} \cdot \frac{180}{\pi} = -150$

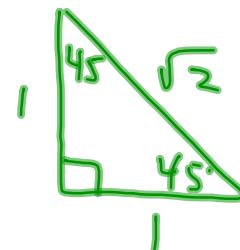reference

Use the reference angle to find a trig value.

Ex 3) sin 135

$$\sin 135° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

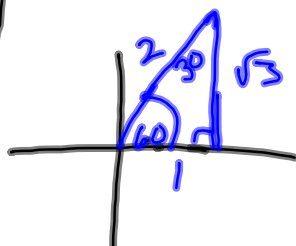Ex 4) cos 210

$$\cos 210° = \boxed{\frac{-\sqrt{3}}{2}}$$

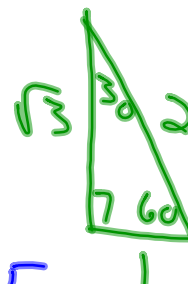Ex 5) cot $\frac{7\pi}{3}$

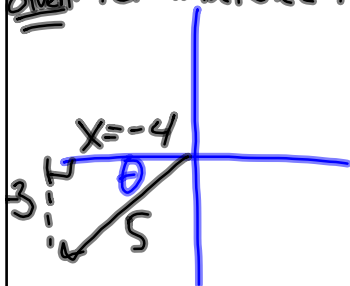$$\frac{7\pi}{3} \times \frac{180}{\pi} = \frac{420}{-360} \over 60°$$

$$\cot \frac{7\pi}{3} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

Ex 6) csc θ = -5/3. Fin the exact value of the remaining trig functions.

Given: terminal side in Quad III

$$\csc\theta = -\frac{5}{3} = \frac{hyp}{opp} = \frac{r}{y} = \frac{5}{-3}$$

X=-4

$$(-3)^2 + x^2 = 5^2$$
$$9 + x^2 = 25$$
$$x^2 = 16$$
$$x = \pm 4$$

$$\sin\theta = \frac{3}{-5}$$

$$\cos\theta = \frac{-4}{5} \qquad \sec\theta = \frac{5}{-4}$$

$$\tan\theta = \frac{-3}{-4} = \frac{3}{4} \qquad \cot\theta = \frac{4}{3}$$