

3.5 Solving Systems of Equations in Three Variables

Objective: Solve systems of linear equations in 3 variables. Solve real-world problems using systems of linear equations in 3 variables.

**An Ordered Triple : (x, y, z)

Ex1) Solve the system

- ① $5x + 3y + 2z = 2$
- ② $2x + y - z = 5$
- ③ $x + 4y + 2z = 16$

$$\boxed{\begin{matrix} (-2, 6, -3) \\ x \quad y \quad z \end{matrix}}$$

$$\begin{array}{r|l} \begin{array}{l} 1+2"z" \\ 5x+3y+2z=2 \\ 4x+2y-2z=10 \\ \hline 4x+5y=12 \end{array} & \begin{array}{l} 2+3"z" \\ 4x+2y-2z=10 \\ x+4y+2z=16 \\ \hline 5x+6y=26 \end{array} \\ \hline \begin{array}{l} #5 \\ 5(-2)+6y=26 \\ -10+6y=26 \\ +10 \quad +10 \\ \hline 6y=36 \\ y=6 \end{array} & \begin{array}{l} #2 \\ 2(-2)+6-z=5 \\ 2-z=5 \\ -z=3 \\ z=-3 \end{array} \end{array}$$

$$\begin{array}{r} 4+5 \\ 5x+3y=72 \\ -25x-30y=-130 \\ \hline 29x=-58 \\ \frac{29x}{29} = \frac{-58}{29} \\ x=-2 \end{array}$$

- Ex2) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$
- ① $4x - 6y + 4z = 12$
 - ② $6x - 9y + 6z = 18$
 - ③ $5x - 8y + 10z = 20$

$$\begin{array}{r|l} \begin{array}{l} 1+2"x" \\ 12x-18y+12z=36 \\ -12x+18y-12z=-36 \\ \hline 0=0 \text{ True} \\ \text{(Same plane)} \end{array} & \begin{array}{l} 2+3"x" \\ 30x-45y+30z=90 \\ -30x+48y-40z=-20 \\ \hline 3y-30z=-30 \end{array} \\ \hline & \boxed{\text{infinitely Many}} \end{array}$$

- Ex3) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$
- ① $6a + 12b - 8c = 24$
 - ② $9a + 18b - 12c = 30$
 - ③ $4a + 8b - 7c = 26$

$$\begin{array}{r} 1+2"a" \\ 18a+36b-24c=72 \\ -18a-36b+24c=-60 \\ \hline 0=12 \\ \text{Parallel Planes} \end{array}$$

$$\boxed{\text{No Solution}}$$

Problems from the Homework

12. ① $2x - y = 2$
 ② $3z = 21$
 ③ $4x + z = 19$

$$\begin{array}{r} 4x + z = 19 \\ \underline{7 - 7} \\ 4x = 12 \\ \underline{4x = 12} \\ z = 7 \end{array}$$

$$\begin{array}{r} 2(3) - y = 2 \\ \underline{-6 - y = 2} \\ -y = -4 \end{array}$$

$$\boxed{\begin{matrix} (3, 4, 7) \\ x \quad y \quad z \end{matrix}}$$

18. ① $3x + y + z = 4$
 ② $2x + 2y + 3z = 3$
 ③ $x + 3y + 2z = 5$

$$\begin{array}{r} 1 + 2y \\ \underline{-6x - 2y + 2z = -8} \\ 2x + 2y + 3z = 3 \end{array}$$

$$\textcircled{4} -4x + z = -5$$

$$\begin{array}{r} 1 + 3y \\ \underline{-7x - 3y - 3z = -12} \\ x + 3y + 2z = 5 \end{array}$$

$$\textcircled{5} -8x - z = -7$$

$$\begin{array}{r} 4 + 5z \\ \underline{-4x + z = -5} \\ \underline{-8x + z = -7} \\ -12x = -12 \end{array}$$

$$\textcircled{x=1}$$

$$\begin{array}{r} \#5 \\ \underline{-8(1) - z = -7} \\ -8 - z = -7 \\ \underline{+8 \quad +8} \\ -z = 1 \\ z = -1 \end{array} \quad \begin{array}{l} (1, 2, -1) \\ \#1 \\ \underline{3(1) + y + (-1) = 4} \\ 2 + y = 4 \\ y = 2 \end{array}$$