

4-5 Determinants

Objective: Evaluate the determinant of 2x2 & 3x3 matrix

A Determinant is a square array of numbers or variables enclosed between two parallel lines.

*Need *
to
Memorize*

To Evaluate a Second Order Determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

I. Find the value of each determinant

$$\text{Ex1) } \begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix} = 6 \cdot 0 - 4 \cdot (-1) = 0 - (-4) = \textcircled{4}$$

$$\begin{aligned} \text{Ex2) } \begin{vmatrix} -6 & 7 \\ -9 & 3 \end{vmatrix} &= -6 \cdot 3 - 7 \cdot (-9) = -18 - (-63) \\ &= -18 + 63 \\ &= \textcircled{45} \end{aligned}$$

To Evaluate a Third-Order Determinant

Way one: Expansion of minors

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\begin{aligned} \text{Ex 3) } \begin{vmatrix} 2 & 7 & -3 \\ -1 & 5 & -4 \\ 6 & 9 & 0 \end{vmatrix} &= 2 \begin{vmatrix} 5 & -4 \\ 9 & 0 \end{vmatrix} - 7 \begin{vmatrix} -1 & -4 \\ 6 & 0 \end{vmatrix} + (-3) \begin{vmatrix} -1 & 5 \\ 6 & 9 \end{vmatrix} \\ &= 2(0 - (-36)) - 7(0 - (-24)) + (-3)(-9 - 30) \\ &= 2(36) - 7(24) + (-3)(-39) \\ &= 72 - 168 + 117 = \textcircled{21} \end{aligned}$$

$$\begin{aligned} \text{Ex 4) } \begin{vmatrix} 3 & -2 & -1 \\ -1 & 0 & 1 \\ -5 & 2 & 1 \end{vmatrix} &= 3 \begin{vmatrix} -1 & 0 \\ -5 & 2 \end{vmatrix} + 2 \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -1 \\ 0 & -5 \end{vmatrix} \\ &= 3(-2 - 0) + 2(4 - 0) + (-1)(10 - 0) \\ &= 3(-2) + 2(4) + (-1)(10) \\ &= -6 + 8 + 10 = \textcircled{12} \end{aligned}$$

Way two: Using Diagonals

Bottom - Top

Ex 5)
$$\begin{array}{ccc|ccc} 2 & 7 & -3 & 2 & 7 & \\ -1 & 5 & -4 & -1 & 5 & \\ 6 & 9 & 0 & 6 & 9 & \end{array}$$

$$\underline{-90} + \underline{-72} + \underline{0} = \underline{-162}$$

$$\underline{0} + \underline{168} + \underline{27} = \underline{-141}$$

$$-141 - (-162) = \boxed{21}$$

Ex 6)
$$\begin{array}{ccc|ccc} 3 & -2 & -1 & 3 & -2 & \\ 2 & -1 & 0 & 2 & -1 & \\ 0 & -5 & 2 & 0 & -5 & \end{array}$$

$$\underline{0} + \underline{0} + \underline{-8} = \underline{-8}$$

$$\underline{-6} + \underline{0} + \underline{10} = \underline{4}$$

$$4 - (-8) = \textcircled{12}$$

* Test Question!

Area of a Triangle

If (a, b), (c, d), and (e, f) are vertices, then

$$A = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

it's possible to be (-); however
area is always (+), so make the answer
(+) at the end!

Ex 7) Find the area of a triangle whose vertices are (0, -1), (-2, -6) and (3, -2).

$$A = \frac{1}{2} \begin{vmatrix} 0 & -1 & 1 \\ -2 & -6 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

Save for later

$$= 0 \begin{vmatrix} -6 & 1 \\ -2 & 1 \end{vmatrix} + 1 \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} -2 & -6 \\ 3 & -2 \end{vmatrix}$$

$$= 0 + 1(-2-3) + 1(4-18)$$

$$= -5 + 22$$

$$\frac{1}{2}(17) = \textcircled{8.5 \text{ units}^2}$$