4-7 Identity and Inverse Matrices

Objective: Determine whether 2 matrices are inverses.
Find the inverse of a 2 x 2.

Identity Matrix

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Read page 195... $\boxed{A \cdot A^{-1} = A^{-1} \cdot A = I}$

Determine whether each pair of matrices are inverses.

$xy = yx = I_2$

Ex 1) $x = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ *and* $y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$xy = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3+-2 & 6+-6 \\ -1+1 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$yx = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3+-2 & -2+2 \\ 3+-3 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $xy = yx = I_2$
These matrices are inverses of each other.

Ex 2) $P = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$ *and* $Q = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$PQ = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3+-2 & -9+-4 \\ \_\_ & \_\_ \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ & \end{bmatrix}$

Since $-13 \neq 0$
These two matrices are inverses of each other.

Some matrices do not have an inverse. You can determine whether it does or does not by using the determinant...

$\boxed{ad - bc = \det A}$ (determinant of A)...If the determinant of A is 0, then the matrix does <u>**not**</u> have an inverse.

<u>**How to find an Inverse of a 2x 2.**</u>

$$A=\begin{bmatrix}3&-1\\4&-2\end{bmatrix} \text{ then } A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$

determinant

Memorize

$$\begin{bmatrix}a&b\\c&d\end{bmatrix}$$

<u>**Find the inverse.**</u>

<u>Ex 3)</u> $R=\begin{bmatrix}-4&-3\\8&6\end{bmatrix}$

$\det R = -24 - -24 = -24 + 24 = \boxed{0}$ Inverse does not exist!

Ex 4) $P=\begin{bmatrix}3&1\\5&2\end{bmatrix}$

$\det P = 6 - 5 = 1$

$\frac{1}{1}\begin{bmatrix}2&-1\\-5&3\end{bmatrix} = \begin{bmatrix}2&-1\\-5&3\end{bmatrix} = P^{-1}$

Ex 5) $S=\begin{bmatrix}-1&0\\8&-2\end{bmatrix}$

$\det S = 2 - 0 = 2$

$\frac{1}{2}\begin{bmatrix}-2&0\\-8&-1\end{bmatrix} = \begin{bmatrix}-1&0\\-4&-\frac{1}{2}\end{bmatrix} = S^{-1}$