

5-7 Rational Exponents

Objective: Write expressions with rational exponents in radical form and vice versa.

Simplify expressions in exponential or radical form.

a ^{Power}/_{index}

Key Ideas

$$b^{1/n} = n\sqrt[n]{b}$$

$$b^{m/n} = n\sqrt[n]{b^m} = (n\sqrt[n]{b})^m$$

$$b^{m/n} = (b^m)^{1/n} = (b^{1/n})^m$$

$$a^{5/6} = \sqrt[6]{a^5}$$

$$= (\sqrt[6]{a})^5$$

$$a^{5/6} = (a^5)^{1/6}$$

$$= (a^{1/6})^5$$

I. Radical Form - Write each expression in radical form.

EX 1. $a^{1/4}$

$$\sqrt[4]{a^1}$$

EX 2. $a^{5/6}$

$$\sqrt[6]{a^5}$$

EX 3. $m^{3/2}$

$$\sqrt{m^3}$$

II. Exponential Form - Write each radical using rational exponents.

EX 4. $\sqrt[5]{b}$

$$b^{1/5}$$

Power
Index

EX 5. $\sqrt{w^5}$

$$w^{5/2}$$

EX 6. $8\sqrt[3]{c}$

$$8c^{3/8}$$

III. Evaluate

EX 7. $49^{-1/2}$

$$= \frac{1}{49^{1/2}} = \frac{1}{\sqrt{49}} = \left(\frac{1}{7}\right)$$

Recall

$$5^{-2} = \frac{1}{5^2}$$

EX 8. $32^{2/5}$

$$= \sqrt[5]{32^2} = (\sqrt[5]{32})^2 = (2)^2 = (4)$$

EX 9. $243^{3/5}$

$$= \sqrt[5]{243^3} = (\sqrt[5]{243})^3 = 3^3 = (27)$$

IV. Simplify

-Read P.260

-If you start the problem with radicals, answer with radicals. If you start with rational exponents, answer with rational exponents.

EX 10. $x^{1/5} \cdot x^{7/5} = x^{8/5} = \sqrt[5]{x^8} = \sqrt[5]{x^5} \sqrt[5]{x^3}$

EX 11. $y^{-3/4} = \frac{1}{y^{3/4}} = \frac{1}{\sqrt[4]{y^3}} \cdot \frac{\sqrt[4]{y}}{\sqrt[4]{y}} = \frac{\sqrt[4]{y}}{\sqrt[4]{y^4}} = \frac{\sqrt[4]{y}}{y}$

OR $X \cdot X^{3/5}$

OR $\frac{y^{1/4}}{y}$

Hints:

-To reduce the index, look for perfect squares, cubes, 4ths, then rewrite and

simplify.

EX 12. $\frac{\sqrt[8]{81}}{\sqrt[6]{3}} = \frac{81^{1/8}}{3^{1/6}} = \frac{(3^4)^{1/8}}{3^{1/6}} = \frac{3^{4/8}}{3^{1/6}} = \frac{3^{1/2}}{3^{1/6}} = 3^{1/2 - 1/6} = 3^{1/3}$

either answer is ok

EX 13. $\sqrt[6]{4x^4} = 4^{1/6} \cdot x^{4/6} = (2^{2/6}) \cdot x^{2/3} = 2^{1/3} \cdot x^{2/3} = \sqrt[3]{2} \cdot x^{2/3} = \sqrt[3]{2x^2}$

$= 3^{1/3}$
or
 $\sqrt[3]{3}$

EX 14. $4\sqrt[4]{9z^2}$

$= 9^{1/4} \cdot z^{2/4}$
 $= (3^2)^{1/4} \cdot z^{1/2}$
 $= 3^{2/4} \cdot z^{1/2}$

$= 3^{1/2} \cdot z^{1/2}$ or $\sqrt{3z}$