

## 5-8 Radical Equations and Inequalities

**Objective:** Solve equations and inequalities containing radicals.

Isolate the radical!!

## I. Radical Equations

EX 1.  $\sqrt{x+1} + 2 = 4$

$$\begin{array}{r} \sqrt{x+1} + 2 = 4 \\ \underline{-2 \quad -2} \\ \sqrt{x+1} = 2 \\ \underline{x+1 = 4} \\ \underline{-1 \quad -1} \\ x = 3 \end{array}$$

Check

$$\begin{array}{r} \sqrt{3+1} + 2 = 4 \\ \sqrt{4} + 2 = 4 \\ 2 + 2 = 4 \\ 4 = 4 \\ \checkmark \end{array}$$

EX 2.  $\sqrt{y-2} - 1 = 5$

$$\begin{array}{r} \sqrt{y-2} - 1 = 5 \\ \underline{+1 \quad +1} \\ \sqrt{y-2} = 6 \\ \underline{y-2 = 36} \\ \underline{+2 \quad +2} \\ y = 38 \end{array}$$

EX 3.  $(\sqrt{x-15})^2(3-\sqrt{x})^2(3-\sqrt{x})$

$$\begin{array}{r} \sqrt{x-15} = 9 - 6\sqrt{x} + \cancel{x} \\ \underline{-x} \\ -15 = 9 - 6\sqrt{x} \\ \underline{-9 \quad -9} \\ -24 = -6\sqrt{x} \\ \underline{-6 \quad -6} \\ 4 = (\sqrt{x})^2 \\ 16 = x \end{array}$$

Check

$$\begin{array}{r} \sqrt{16-15} = 3 - \sqrt{16} \\ \sqrt{1} = 3 - 4 \\ 1 \neq -1 \end{array}$$

Check

$$\begin{array}{r} \sqrt{38-2} - 1 = 5 \\ \sqrt{36} - 1 = 5 \\ 6 - 1 = 5 \\ 5 = 5 \checkmark \end{array}$$

Does not check!  
No Solution

## II. Cube Root Equations

EX 4.  $3\sqrt[3]{5n-1} - 2 = 0$

$$\begin{array}{r} 3\sqrt[3]{5n-1} - 2 = 0 \\ +2 \quad +2 \\ \hline 3\sqrt[3]{5n-1} = 2 \\ \frac{3}{3} \sqrt[3]{5n-1} = \frac{2}{3} \end{array}$$

$$\begin{array}{r} (\sqrt[3]{5n-1})^3 = (\frac{2}{3})^3 \\ 5n-1 = \frac{8}{27} \\ +1 \quad +1 \\ \hline 5n = \frac{35}{27} \end{array}$$

$$\begin{array}{r} 5n = \frac{35}{27} \\ \frac{1}{5} \cdot 5n = \frac{35}{27} \cdot \frac{1}{5} \\ \hline n = \frac{7}{27} \end{array}$$

Checks!

EX 5.  $(3y+1)^{1/3} + 5 = 0$

$$\begin{array}{r} \sqrt[3]{3y+1} + 5 = 0 \\ -5 \quad -5 \\ \hline (\sqrt[3]{3y+1})^3 = (-5)^3 \end{array}$$

$$\begin{array}{r} 3y+1 = -125 \\ -1 \quad -1 \\ \hline 3y = -126 \end{array}$$

$$\begin{array}{r} \frac{3y}{3} = \frac{-126}{3} \\ \hline y = -42 \end{array}$$

Checks!

## III. Radical Inequalities

EX 6.  $2 + \sqrt{4x-4} \leq 6$

$$\begin{array}{r} \sqrt{4x-4} \leq 4 \\ -2 \quad -2 \end{array}$$

$$\begin{array}{r} 4x-4 \leq 16 \\ +4 \quad +4 \\ \hline 4x \leq 20 \\ \frac{4x}{4} \leq \frac{20}{4} \end{array}$$

$$\begin{array}{r} x \leq 5 \end{array}$$

**But** you can't take the square root of a negative so...

$$\begin{array}{r} 4x-4 \geq 0 \\ 4x \geq 4 \\ \hline x \geq 1 \end{array}$$

EX 7.  $\sqrt{3x-6} + 4 \leq 7$

$$\begin{array}{r} \sqrt{3x-6} \leq 3 \\ -4 \quad -4 \end{array}$$

$$\begin{array}{r} 3x-6 \leq 9 \\ +6 \quad +6 \\ \hline 3x \leq 15 \\ x \leq 5 \end{array}$$

**But**  $3x-6 \geq 0$

$$\begin{array}{r} 3x-6 \geq 0 \\ +6 \quad +6 \\ \hline 3x \geq 6 \\ \frac{3x}{3} \geq \frac{6}{3} \\ \hline x \geq 2 \end{array}$$

Final Answer



$$x \mid 1 \leq x \leq 5$$

$$x \mid 2 \leq x \leq 5$$