6.5 The Quadratic Formula & The Discriminant

Objective: Solve quadratic equations by using the quadratic formula.
Use the discriminant to determine the number and type of solutions.

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

I. Solve using the Quadratic Formula

EX 1. $x^2 - 8x = 33$

$x^2 - 8x - 33 = 0$

$a = 1$, $b = -8$, $c = -33$

$x = \dfrac{8 \pm \sqrt{64 - 4 \cdot 1 \cdot -33}}{2(1)}$

$x = \dfrac{8 \pm \sqrt{196}}{2} = \dfrac{8 \pm 14}{2}$

$x = \dfrac{8+14}{2} = \dfrac{22}{2} = 11$

$x = \dfrac{8-14}{2} = \dfrac{-6}{2} = -3$

EX 2. $x^2 - 34x + 289 = 0$

$a = 1$, $b = -34$, $c = 289$

$x = \dfrac{34 \pm \sqrt{1156 - 4 \cdot 1 \cdot 289}}{2(1)}$

$x = \dfrac{34 \pm \sqrt{0}}{2}$

$x = \dfrac{34 \pm 0}{2} = \dfrac{34}{2} = 17$

One Solution

$a = 2$
$b = 4$
$c = -5$

EX 3. $2x^2 + 4x - 5 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 4 \cdot 2 \cdot (-5)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{56}}{4}$$

$$x = \frac{-4 \pm \sqrt{4}\sqrt{14}}{4}$$

$$x = \frac{-4 \pm 2\sqrt{14}}{4}$$

$$x = \frac{-2 \pm \sqrt{14}}{2}$$

$a = 1$
$b = -4$
$c = 13$

EX 4. $x^2 - 4x = -13$

$$x^2 - 4x + 13 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 13}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{36}}{2} = \frac{4 \pm 6i}{2}$$

$$x = 2 \pm 3i$$

Discriminant- Tells how many and what kind of solutions.

$b^2 - 4ac > 0$ 2 real roots (rational/irrational)

$b^2 - 4ac = 0$ 1 real root

$b^2 - 4ac < 0$ No real roots, 2 complex (imaginary).

NOTE: if $b^2 - 4ac > 0$ and it is a perfect square: rational
and it is not a perfect square: irrational

x intercepts / solutions

II. How many roots and what type?

EX 5. $x^2 + 6x + 9 = 0$

$a=1\ b=6\ c=9$

1 real sol.

$b^2-4ac=36-4(1)(9)$

$=36-36=0$

EX 6. $x^2 + 3x + 5 = 0$

$a=1, b=3, c=5$

$b^2-4ac=9-4(1)(5)$

$=9-20=-11$

discriminant

2 imaginary sol.

$a=1\ b=4\ c=4$

EX7. $x^2 + 8x - 4 = 0$

$b^2-4ac=(4)^2-4(1)(-4)$

$=80$

discriminant

2 real solutions