

6.6 Analyzing Graphs of Quadratic Functions

Objective: Analyze quadratic functions of the form

$y = a(x - h)^2 + k$ and write a quadratic in vertex form.

Explore: $y = x^2$ is called a parent function.

$$y = ax^2 + bx + c$$

Graph and discuss:

- | | | |
|------------------|--------------------|-------------------------|
| 1. $y = x^2 + 3$ | 1. $y = (x - 2)^2$ | 1. $y = 5(x - 2)^2$ |
| 2. $y = x^2 - 6$ | 2. $y = (x + 6)^2$ | 2. $y = -4(x + 6)^2$ |
| | | 3. $y = (1/3)(x + 3)^2$ |

Vertex Form: $y = a(x - h)^2 + k$

Horizontal translation points to $(x - h)$

Vertical Translation points to $+ k$

$a > 0$, opens up

$a < 0$, opens down

$|a| > 1$, narrower graph of $y = x^2$

$|a| < 1$, wider graph $y = x^2$

(h, k) = vertex

$x = h$ Line of symmetry

I. Write in vertex form, then ID vertex, axis of symmetry, and direction of opening.

EX 1. $y = x^2 + 8x - 5$

$$y = (x^2 + 8x + 16) - 5 - 16$$

$$y = 1(x+4)^2 + 21$$

$$y = a(x-h)^2 + k$$

Vertex: $(h,k) = (-4, 21)$

Axis: $x = h, x = -4$

Direction: $a > 0 \rightarrow$ up

vertex form

EX 2. $y = x^2 + 2x + 4$

$$y = (x^2 + 2x + 1) + 4 - 1$$

$$y = (x+1)^2 + 3$$

$$y = a(x-h)^2 + k$$

Vertex: $(h,k) = (-1, 3)$

Axis: $x = h, x = -1$

Direction: $a > 0, \text{ up}$

vertex form

EX 3. $y = -3x^2 + 6x - 1$

$$y = -3(x^2 - 2x + 1) - 1 + 3$$

$$y = -3(x-1)^2 + 2$$

$$y = a(x-h)^2 + k$$

Vertex: $(h,k) = (1, 2)$

Axis: $x = h, x = 1$

direction: $a = -3; 3 < 0$ down

vertex form

EX 4. $y = -2x^2 - 4x + 2$

$$y = -2(x^2 + 2x + 1) + 2 + 2$$

$$y = -2(x+1)^2 + 4$$

Vertex: $(h,k) = (-1, 4)$

Axis: $x = -1$

direction: down

II. Write an equation in vertex form given the following points

EX 5. Vertex: $(1, 2)$
 Point: $(3, 4)$

$$y = a(x-h)^2 + k$$

$$y = \frac{1}{2}(x-1)^2 + 2$$

$$4 = a(3-1)^2 + 2$$

$$4 = 4a + 2$$

$$\frac{-2}{4} = \frac{4a}{4} \quad a = \frac{1}{2}$$

EX 6. Vertex: $(-1, 4)$
 Point: $(2, 1)$

$$y = a(x-h)^2 + k$$

$$y = -\frac{1}{3}(x+1)^2 + 4$$

$$1 = a(2+1)^2 + 4$$

$$1 = 9a + 4$$

$$\frac{-3}{9} = \frac{9a}{9} \quad a = -\frac{1}{3}$$

⑮ $y = -8x^2 + 3$

$$y = -8(x-0)^2 + 3$$