7.5 Roots and Zeros

Objective: Determine the # and types of roots for a polynomial equation.
Find the zeros of a polynomial function.

Skip 34
36
42

Fundamental Theorem of Algebra pg. 371

- Every polynomial equation of degree greater than zero has at least 1 root in the set of complex numbers.

I. Determine the # and Type of Roots

Ex1) $a - 10 = 0$

$+10 \; +10$

$\boxed{a=10}$ solution

1 solution, real

Ex2) $x^2 + 2x - 48 = 0$

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$(x-6)(x+8)=0$

$x-6=0$ $\quad$ $x+8=0$

$\boxed{x=6}$ $\quad$ $\boxed{x=-8}$

2 solutions, real

Ex3) $3a^3 + 18a = 0$

$3a(a^2+6)=0$

$3a=0$ $\quad$ $a^2+6=0$

$\boxed{a=0}$ $\quad$ $\sqrt{a^2}=\sqrt{-6}$

$\boxed{a=\pm i\sqrt{6}}$

3 solutions, 1 real 2 imaginary

Ex4) $y^4 - 16 = 0$

$(y^2+4)(y^2-4)=0$

$(y^2+4)(y-2)(y+2)=0$

$y-2=0$ $\quad$ $y+2=0$

$\boxed{y=2}$ $\quad$ $\boxed{y=-2}$

$y^2+4=0$

$\sqrt{y^2}=\sqrt{-4}$

$\boxed{y=\pm 2i}$

4 solutions 2 real, 2 imag.

-So p(x) of degree n will have n roots including the imaginary ones.

II. Decartes Rule of Signs: finds # of positive or negative zeros pg.372

Ex5)
Positive: Use P(x)

$$p(x) = x^5 - 6x^4 - 3x^3 + 7x^2 - 8x + 1$$

y n y y y

4 sign changes(y's), so there are 4,2, or 0 posiive real zeros.

Negative: use P(-x)

$$P(-x) = (-x)^5 - 6(-x)^4 - 3(-x)^3 + 7(-x)^2 - 8(-x) + 1$$

$$P(-x) = -x^5 - 6x^4 + 3x^3 + 7x^2 + 8x + 1$$

n y n n n

One sign change(y), so there is 1 negative real zero

Possible Combinations of Zeros

Positive Real	Negative Real	Imaginary	Total
4	1	0	5
2	1	2	5
0	1	4	5

III. Find all zeros of the polynomial (Hint: Use calc to find rational zeros first)

from Graph: $x = 2$

EX 6. $f(x) = x^3 - 4x^2 + 6x - 4$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 6 & -4 \\ & & 2 & -4 & 4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$x^2 - 2x + 2 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = \boxed{1 \pm i}$$

$\boxed{x = 2, \ x = 1 \pm i}$

from calc: $\boxed{x = -1}$

EX 7. $f(x) = x^3 - x^2 + 2x + 4$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 2 & 4 \\ & & -1 & 2 & -4 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$x^2 - 2x + 4 = 0$

$i\sqrt{4}\sqrt{3}$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = \boxed{1 \pm i\sqrt{3}}$$

IV. Write a polynomial given zeros

Ex8) x = 4, 4-i, $4+i$

$p(x) = (x-4)(x-(4-i))(x-(4+i))$

$p(x) = (x-4)\underbrace{(x-4+i)(x-4-i)}$

$p(x) = (x-4)(x^2 - 8x + 17)$

$$\boxed{p(x) = x^3 - 12x^2 + 49x - 68}$$

26. $x = 4$
28. $x = 7$
30. $x = .5$
32. $x = 4, -1$

Ex9) x = 3, 2-i

	x	-4	$+i$
x	x^2	$-4x$	$\cancel{ix}$
-4	$-4x$	16	$\cancel{-4i}$
$-i$	$\cancel{-ix}$	$\cancel{+4i}$	$-i^2 = 1$

$-(-1)$

	x^2	$-8x$	$+17$
x	x^3	$-8x^2$	$17x$
-4	$-4x^2$	$+32x$	-68