

7.7 Operations with Functions Day 2

I. Simplifying Composition of Functions

Ex1) $f(x) = x + 3$ $g(x) = x^2 + x - 1$ Find $(f \circ g)x$ and $(g \circ f)x$.

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(x^2 + x - 1) \\ &= x^2 + x - 1 + 3 \\ \boxed{f \circ g} &= \boxed{x^2 + x + 2} \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(x + 3) \\ &= (x + 3)^2 + (x + 3) - 1 \\ &= x^2 + 6x + 9 + x + 3 - 1 \\ &= \boxed{x^2 + 7x + 11} \end{aligned}$$

Evaluate if $x = 2$.

$$\begin{aligned} &= f \circ g(2) \\ &= 2^2 + 2 + 2 \\ &= \boxed{8} \end{aligned}$$

$$\begin{aligned} g \circ f(2) &= 2^2 + 7(2) + 11 \\ &= \boxed{29} \end{aligned}$$

Ex2) $f(x) = 3x^2 - x + 4$ $g(x) = 2x - 1$ $h(x) = x^2 - 3$

$$\begin{aligned} g \circ h(x) &= g(h(x)) \\ &= g(x^2 - 3) \\ &= 2(x^2 - 3) - 1 \\ &= 2x^2 - 6 - 1 \\ \boxed{g \circ h(x)} &= \boxed{2x^2 - 7} \end{aligned}$$

$$\begin{aligned} h(g(x)) &= \\ h(2x - 1) &= (2x - 1)^2 - 3 = 4x^2 - 4x + 1 - 3 \\ &= \boxed{4x^2 - 4x - 2} \end{aligned}$$

Evaluate $x = -2$

$$\begin{aligned} g(4) &= 2(4) - 1 = 7 \\ g(-1) &= 2(-1) - 1 = -3 \end{aligned}$$

A. $f(g(-1)) = f(-3) = 3(-3)^2 - (-3) + 4 = \boxed{34}$

B. $h(g(4)) = h(7) = 7^2 - 3 = \boxed{46}$ or

C. $(f \circ (h \circ g))(2) = f(h(g(2))) = f(h(3)) = f(6) = 3(6)^2 - 6 + 4 = \boxed{106}$

$$\begin{aligned} g(2) &= 2 \cdot 2 - 1 = 3 \\ h(3) &= 3^2 - 3 = 6 \end{aligned}$$

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$$\begin{aligned} 4(4)^2 - 4(4) - 2 \\ 64 - 16 - 2 \\ \underline{46} \end{aligned}$$

#30 $g(h(x)) = g(-3x+1) = -5(-3x+1) = \boxed{15x-5}$

$$\begin{aligned} g(x) &= -5x \\ h(x) &= -3x+1 \\ h(g(x)) &= h(-5x) = -3(-5x) + 1 = \boxed{15x+1} \end{aligned}$$

Ex3) Tracy has \$100 deducted from every paycheck for retirement before taxes are applied, which reduces her taxable income. Her state income tax rate is 4%. If Tracy earns \$1500 every pay period, find the difference in her net income if she has the retirement deduction before or after state taxes.