7.8 Inverse Functions & Relations

Objective: Find the inverse of a function or relation
Determine whether 2 functions or relations are inverses

I. Relations - set of ordered pairs
Inverse Relation - set of ordered pairs obtained by reversing the coordinates of each original ordered pairs

Ex) Relation: f = {(1,2), (3,4), (5,6)}

f^{-1} = {(2,1), (4,3), (6,5)}

II. Finding Inverses 1)switch x + y 2) Solve for y

Ex1) $\cancel{f(x)}\ y = \frac{x+6}{2}$

① $x = \frac{y+6}{2}$

② $2x = y+6$

$2x - 6 = y$

$\boxed{f^{-1}(x) = 2x-6}$

Ex2) $\cancel{f(x)}\ y = \frac{-1}{2}(x)+1$

① $x = \frac{-1}{2}\cdot y + 1$

② $-2(x-1) = \frac{-1}{2}\cdot y\cdot(-2)$

$-2x+2 = y$

$\boxed{f^{-1}(x) = -2x+2}$

Ex2.5) $y = 10$

① $\boxed{x = 10}$

inverse

Note: f(x) + $f^{-1}(x)$ are reflections over the line y=x.

III. Inverse functions - f(g(x)) = x and g(f(x))=x then the 2 functions are inverses

EX 3. f(x) = 5x + 10 g(x)=(1/5)x - 2

$f(g(x)) = f(\frac{1}{5}x - 2) = 5(\frac{1}{5}x - 2) + 10 = x - 10 + 10 = \boxed{x}$

$g(f(x)) = g(5x+10) = \frac{1}{5}(5x+10) - 2 = x + 2 - 2 = \boxed{x}$

f(x) & g(x) are inverses of each other.

EX 4. f(x) = 6x + 2 g(x) = x - (1/3)

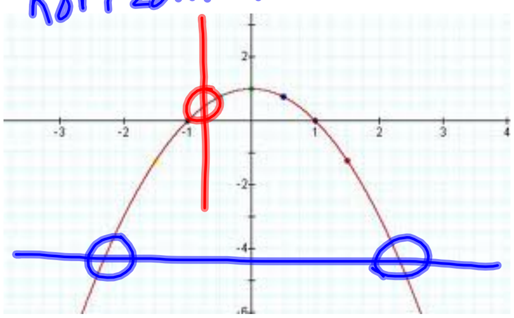
$f(g(x)) = f(x - \frac{1}{3}) = 6(x - \frac{1}{3}) + 2 = 6x - 2 + 2$
$= \boxed{6x} \neq x$

$g(f(x)) =$

So we can say that f(x) & g(x) are <u>not</u> inverses of each other.

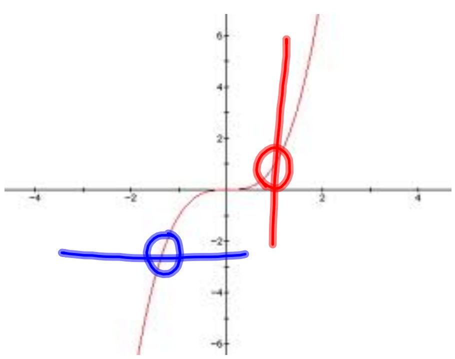
IV. One-to-One Test/Horizontal Line Test: If a function passes the horizontal line test then we say the function is one-to-one which means the function's inverse is a function.

vertical line Test*
horizontal line Test*



Function : yes, it passes vertical line test.

Inverse function: Not a function, since it fails the horizontal line test.



Function : yes, it passes the vertical line test.

Inverse function: yes, the graph also passes the horizontal line test so its inverse is also a function.

*We then can also say the function is one-to-one!