9-3 Graphing Rational Functions

A **Rational Function** is in the form of $f(x) = \frac{p(x)}{q(x)} = \frac{\text{polynomial}}{\text{polynomial}}$

Their graphs may have breaks in continuity such as asymptote or hole in the graph.
See page 485 and 486.

I. Determine the equations of any vertical asymptotes and the values of x for any holes in the graph.

Ex 1) $f(x) = \frac{x^2 - 1}{x^2 - 6x + 5} = \frac{(x+1)(x-1)}{(x-5)(x-1)}$

$x - 5 \neq 0$ $\quad$ $x - 1 \neq 0$

$x \neq 5$ $\quad$ $x \neq 1$

$\boxed{x = 5}$ Vertical asymptote $\quad$ $\boxed{x = 1}$ hole in graph

Ex 2) $f(x) = \frac{x^2 - 4}{x^2 + 5x + 6} = \frac{(x+2)(x-2)}{(x+3)(x+2)}$

$x + 3 \neq 0$ $\quad$ $x + 2 \neq 0$

$x \neq -3$ $\quad$ $x \neq -2$

$\boxed{x = -3}$ V.A. $\quad$ $x = -2$ hole

Key

If the factor in the denominator cancels with a factor in the numerator, there is a hole in the graph. If it does not cancel, there is a vertical asymptote.

II. Graph.

Ex 3) $f(x) = \dfrac{x^2 - 9}{x + 3} = \dfrac{(x+3)(x-3)}{x+3} = x-3$

$\boxed{y=x-3}$

$m=1$
$b=-3$

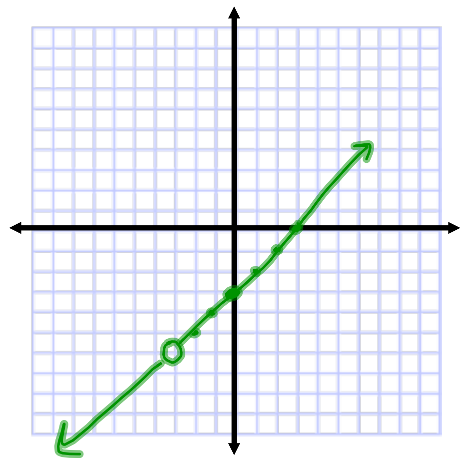
$y=mx+b$

y-int

$x+3 \neq 0$

$\boxed{\begin{array}{c} x=-3 \\ \text{hole} \end{array}}$

Graph this



$\dfrac{0}{0-2}=0$

Ex 4) $f(x) = \dfrac{x}{x - 2}$

$\dfrac{1}{1-2}=-1$

$x-2 \neq 0$

$x \neq 2$

$x=2$ V.A.

$\dfrac{3}{3-2}=3$

$\dfrac{4}{4-2}=2$

x	y
0	0
1	-1
3	3
4