

**9-4 Direct, Joint, and Inverse Variation**

**Objective:** Recognize and solve direct, joint, and inverse variation problems.

I. **Direct:** y varies directly as x if there is some nonzero k such that  $y = kz$ . (k is a constant)

Memorize  $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

example: (height, weight) (x, y)  
 (radius, area) (↑, ↑)  
 (↓, ↓)

Ex 1) If y varies directly as x and  $y_1 = -15$  when  $x_1 = 5$ . Find  $y_2$  when  $x_2 = 3$ .

$\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\frac{5}{-15} = \frac{3}{y_2}$

$\frac{3 \cdot 15}{3} = y_2 = \frac{-45}{3} = -9 = y_2$

II. **Joint Variation:** y varies jointly as x and z if there is some number k such that  $y = kxz$  where k, x, and z cannot be zero.

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2}$

Example:  $V = \pi r^2 h$   
 $y = k \cdot x \cdot z$   
 ↑ r ↑ h ↑

Ex 2) y varies jointly as x and z. Find  $y_1$  when  $x_1 = 10$ ,  $z_1 = 5$ , if  $y_2 = 12$  when  $z_2 = 8$  and  $x_2 = 3$ .

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2}$

$\frac{y_1}{10 \cdot 5} = \frac{12}{3 \cdot 8}$

$\frac{y_1}{50} = \frac{12}{24}$

$\frac{y_1}{50} = \frac{1}{2}$

$\frac{50 \cdot 1}{2} = y_1 = 25$

III. Inverse Variation: y varies inversely as x if there is some nonzero constant k such that  $y = k/x$ .

$$\frac{x_1}{y_2} = \frac{x_2}{y_1}$$

Example:  
(Speed, Time)

$$y = \frac{k}{x}$$

Ex 3) If y varies inversely as x and y = -6, when x = 2, find y when x = -7.

$$\frac{x_1}{y_2} = \frac{x_2}{y_1} \quad \frac{2}{y_2} = \frac{-7}{-6}$$

$$\frac{2 \cdot 6}{-7} = y_2 = \left(\frac{12}{-7}\right)$$

⑭  $\frac{n}{m} = 1.5$   
 $n = 1.5m$   
 $k = 1.5 \frac{m}{n}$   
Direct  $\frac{1}{3} \mid \frac{1.5}{4.5} \uparrow$

⑮  $a = 5bc$   
 $k = 5$   
joint

⑯  $p = \frac{12}{q}$   
 $k = 12$   
inverse  $\frac{8}{p} \mid \frac{12}{2} \mid 6 \downarrow$

⑰  $3 = \frac{a}{b}$   
 $a = 3b$   
 $k = 3$   
direct  $\frac{b}{a} \mid \frac{2}{6} \uparrow$

⑳  $V = \frac{1}{3}Bh$   
 $k = \frac{1}{3}$   
joint

Now, let us get started on your homework!!