Advanced Algebra
Chapter 10 Outline

10-1
22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 57, 58, 59, 60, 62, 63, 64 (24)

10-2
Day 1: worksheet
Day 2: 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 68, 69, 70 (26)

10-3
14, 16, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 38, 39, 40, 49, 51, 53, 55 (23)

WKSTS

Quiz on 10-1, 10-2, and 10-3

10-4
18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 45, 46, 47, 52, 53, 54, 55, 60, 61, 63, 66, 67 (25)

10-5
20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 54, 55, 56 (25)

10-6
10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 26, 27, 28, 29, 30 (15)

Quiz on 10-4 and 10-5/ Review Pages 566-570 2-64 Even

Review
Page 571, 1-30

Chapter 10 Test

10-1 Exponential Functions

Objective: Graph exponential functions.
Solve exponential equations and inequalities.

I. Intro: Discuss NCAA Women's Tourney P.523.

graph $y=2^x$ then state domain:
range:

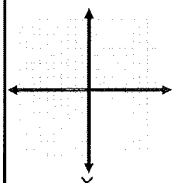
Then graph $y=3^x$, $y=(1/3)^x$, $y=-1(2^x)$ and discuss.

Exponential Functions: $\boxed{y=ab^x}$ where a does not equal 0, b>0, and b does not equal 1.

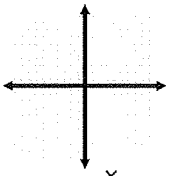
Characteristics

1. Function is continuous and one to one.
2. D: all real numbers.
3. x- axis is an asymptote.
4. R: all positive numbers if a>0.
   R: all negative numbers if a<0.
5. (0,a) is a point where a is the y-intercept.
6. $y=ab^x$ and $y=a(1/b)^x$ are reflections across the y-axis.

II. Growth vs. Decay



$y=ab^x$
a>0, b>1



$y=ab^x$
a>0, 0<b<1

Growth or Decay, or Neither.

Ex 1. $y=(1/5)^x$

$y=3(4)^x$

$y=7(1.2)^x$

EX 2. In Dec. of 1990, there were 5,283,000 cell phone subscribers while in Dec of 2000 there were 109,478,000.

a. Write an exponential function to model this.

III. Simplify

EX 3. $2^{\sqrt{5}} \times 2^{\sqrt{3}}$

EX 4. $(6^{\sqrt{5}})^{\sqrt{6}}$

EX 5. $5^{\sqrt{3}} / 5^{\sqrt{2}}$

IV. Solve

EX 6. $3^{2n+1}=81$

EX 7. $4^{9n-2}=256$

EX 8. $3^{5x}=9^{2x-1}$

EX 9. $4^{3p-1} > (1/256)$

EX 10. $5^{3-2k} > (1/625)$

10-2 Logarithms and Log Functions Day 1

Objective: Evaluate logarithmic expressions.
Solve logarithmic equations and inequalities.

I. **Intro**- Read P.531 **Why is a logarithmic scale used?**

II. **Logarithm**: In general, the inverse of $y=b^x$ is $x=b^y$ (reflections across line y=x). In $x=b^y$, y is called the logarithm of x and written as $y=\log_b x$ (read y equals the log of base b of x).

III. **Log to exponential form** .

EX 1. $\log_3 9 = 2$

EX 2. $\log_{10}(1/100) = -2$

EX 3. $\log_2(1/16) = -4$

IV. **Exponential to log form.**

EX 4. $5^3=125$

EX 5. $27^{1/3}=3$

V. **Evaluate logs** .

EX 6. $\log_3 243$

EX 7. $\log_2 64$.

VI. **Characteristics of logs.**

1. continuous and one to one
2. D: all positive real numbers
3. y-axis is an asymptote
4. R: all real numbers
5. Graph contains the point (1,0) (x-intercept)

NAME _____ DATE _____ PERIOD ____

10-2 Practice

Logarithms and Logarithmic Functions

Write each equation in logarithmic form.

1. $5^3 = 125$
2. $7^0 = 1$
3. $3^4 = 81$
4. $3^{-4} = \frac{1}{81}$
5. $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$
6. $7776^{\frac{1}{5}} = 6$

Write each equation in exponential form.

7. $\log_6 216 = 3$
8. $\log_2 64 = 6$
9. $\log_3 \frac{1}{81} = -4$
10. $\log_{10} 0.00001 = -5$
11. $\log_{25} 5 = \frac{1}{2}$
12. $\log_{32} 8 = \frac{3}{5}$

Evaluate each expression.

13. $\log_3 81$
14. $\log_{10} 0.0001$
15. $\log_2 \frac{1}{16}$
16. $\log_{\frac{1}{3}} 27$
17. $\log_9 1$
18. $\log_8 4$
19. $\log_7 \frac{1}{49}$
20. $\log_6 6^4$
21. $\log_3 \frac{1}{3}$
22. $\log_4 \frac{1}{256}$
23. $\log_9 9^{(n+1)}$
24. $2^{\log_2 32}$

Solve each equation or inequality. Check your solutions.

25. $\log_{10} n = -3$
26. $\log_4 x > 3$
27. $\log_4 x = \frac{3}{2}$
28. $\log_{\frac{1}{5}} x = -3$
29. $\log_7 q < 0$
30. $\log_6 (2y + 8) \geq 2$
31. $\log_y 16 = -4$
32. $\log_n \frac{1}{8} = -3$
33. $\log_b 1024 = 5$
34. $\log_8 (3x + 7) < \log_8 (7x + 4)$
35. $\log_7 (8x + 20) = \log_7 (x + 6)$
36. $\log_3 (x^2 - 2) = \log_3 x$

37. **SOUND** Sounds that reach levels of 130 decibels or more are painful to humans. What is the relative intensity of 130 decibels?

38. **INVESTING** Maria invests \$1000 in a savings account that pays 8% interest compounded annually. The value of the account A at the end of five years can be determined from the equation $\log A = \log[1000(1 + 0.08)^5]$. Find the value of A to the nearest dollar.

Day 2 on 10-2

VII. Properties of logs.

EX 8. $\log_6 6^8$

EX 9. $3^{\log_3(4x-1)}$

VIII. Solving log equations.

EX 10. $\log_8 n=(4/3)$

EX 11. $\log_5(p^2-2)=\log_5 p$

EX 12. $\log_4 x^2=\log_4(4x-3)$.

IX. Solving inequalities.

If $b>1$, $x>0$, and $\log_b x > y$, then $x > b^y$.

If $b>1$, $x>0$, and $\log_b x < y$, then $0 < x < b^y$.

EX 13. $\log_5 x < 2$.

EX 14. $\log_6 x > 3$.

EX 15. $\log_{10}(3x-4) < \log_{10}(x+6)$.

EX 16. $\log_7(2x+8) > \log_7(x+5)$.

10-3 Properties of Logarithms

Objective: Simplify and evaluate expressions using the properties of logarithms.
Solve logarithmic equations using the properties of logs.

I. Properties

1. Product: $\log_b mn = \log_b m + \log_b n$
2. Quotient: $\log_b (m/n) = \log_b m - \log_b n$
3. Power: $\log_b m^p = p \times \log_b m$

EX 1. Use $\log_5 2 = .4307$ to approximate the value of $\log_5 250$.

EX 2. Use $\log_6 8 = 1.106$ and $\log_6 32 = 1.9343$ to approximate the value of $\log_6 4$.

EX 3. Given $\log_5 6 = 1.1133$, approximate the value of $\log_5 216$.

II. Solve.

EX 4. $4\log_2 x - \log_2 5 = \log_2 125$

EX 5. $\log_8 x + \log_8 (x-12) = 2$

EX 6. $3\log_5 x - \log_5 4 = \log_5 16$

EX 7. $\log_4 x + \log_4 (x-6) = 2$

10-4 Common Logarithms

Objective: Solve exponential equations and inequalities using common logs.
Evaluate logarithmic expressions using the Change of Base Formula.

I. Use a calculator to evaluate each expression to four decimal places.

EX 1. log 5

EX 2. log 7.2

II. Solve.

Ex 3. $3^x=11$

EX 4. $5^{x+2}=62$

EX 5. $2^{7x} > 3^{5x-3}$

EX 6. $5^{3y} < 8^{y-1}$

III. Change of Base Formula (to base 10).

$$\log_a n = \frac{\log n}{\log a}$$

EX 7. $\log_5 12$

EX 8. $\log_4 25$

10-5 Base e and Natural Logarithms

Objective: Evaluate expressions involving the natural base and natural logs.
Solve exponential equations and inequalities using natural logs.

I. Intro- P.554 What is e?

If $y=\ln x$, then $e^y=x$.

II. Evaluate- 4 decimal places.

Ex 1. $e^{.5}$

EX 2. e^{-8}

EX 3. $\ln 3$

Ex 4. $\ln (1/4)$

III. Write equivalent expressions.

EX 5. $e^x=23$

Ex 6. $\ln x=1.2528$

IV. Inverse Property of Base e and natural logs.

$e^{\ln x} = x$ and $\ln e^x = x$

EX 7. $e^{\ln 21}$

Ex 8. $\ln e^{x^2-1}$

V. Solve (all prop. of logs apply to natural logs as well).

EX 9. $3e^{-2x} + 4 = 10$

EX 10. $\ln 3x = .5$

EX 11. $\ln(2x-3) < 2.5$

EX 12. $700 is deposited into an account paying 6% annual interest compounded continuously.

$A=Pe^{rt}$ A=amount after t years, P=principal.
r=rate, t=years.

A. What's the balance after 8 years?

B. How long will it take to grow to a least $2000?

10-6 Exponential Growth and Decay

Objective: Use logs to solve problems involving exponential decay and growth

I. **Exponential decay of the form** $y = a(1 - r)^t$

Example 1: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for 90% of this caffeine to be eliminated from a person's body?

I. Exponential Decay of the form $y = ae^{-kt}$

Example 2: The half-life of Sodium-22 is 2.6 years. The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated.

a) What is the value of k for Sodium-22?

b) A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth?

I. Exponential Growth of the form $y = a(1 + r)^t$

Example 3: The population of a city of one million is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled?

I. Exponential Growth of the form $y = ae^{kt}$

Example 4: As of 2000, Nigeria had an estimated population of 127 million people and the United States had an estimated population of 278 million people. The populations of Nigeria and the United States can be modeled by $N(t) = 127e^{.026t}$ and $U(t) = 278e^{.0009t}$, respectively. According to these models, when will Nigeria's population be more than the population of the United States?