

13-5 Law of Cosines

Objective:

Solve problems by using the Law of Cosines

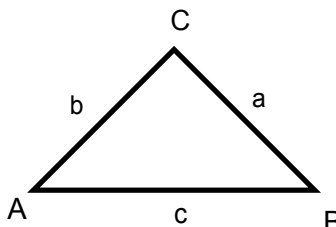
Determine whether a triangle can be solved by first using the law of sines or cosines.

Law of Cosines

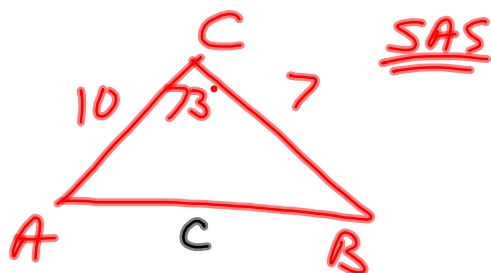
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2~~ac~~bc \cos C$$



Ex 1) Solve triangle ABC.  $C = 73^\circ$ ,  $b = 10$ ,  $a = 7$



$$C = 10.4$$

$$A = 40^\circ$$

$$B = 67^\circ$$

$$\begin{array}{r} 180 \\ - 73 \\ \hline 107 \end{array}$$

$$c^2 = 10^2 + 7^2 - 2 \cdot 10 \cdot 7 \cos 73$$

$$c^2 = 108.1 \dots$$

$$c = 10.4$$

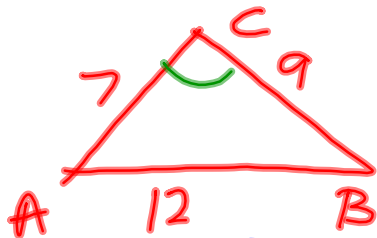
$$\frac{\sin 73}{10.4} = \frac{\sin B}{10}$$

$$\sin^{-1}\left(\frac{10 \sin 73}{10.4}\right) = B = 67^\circ$$

\*Important

When you switch to law of Sines, make sure you find the largest angle last by using  $\triangle$  sum!

Ex 2) Solve triangle ABC.  $a = 9, b = 7, c = 12$



SSS

\* Start w/ largest angle since using law of cosines.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$144 = 81 + 49 - 126 \cos C$$

$$14 = -126 \cos C$$

$$\frac{14}{-126} = \cos C \quad \cos^{-1}\left(\frac{14}{-126}\right) = 96^\circ$$

$$\frac{\sin A}{9} = \frac{\sin 96}{12}$$

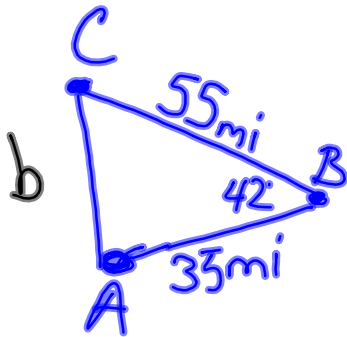
$$A = \sin^{-1}\left(\frac{9 \sin 96}{12}\right)$$

$$A = 48^\circ$$

$$\begin{cases} c = 96 \\ A = 48^\circ \\ B = 36^\circ \end{cases}$$

$$\begin{matrix} 180 \\ - 96 \\ \hline 84 \\ - 48 \\ \hline 36 \end{matrix}$$

Ex 3) A helicopter flies 55 miles from its base at point C to an accident at point B, then 35 miles to the hospital at point A. Angle B equals 42 degrees. How far will the helicopter have to fly to return to its base from the hospital?



$$b^2 = 55^2 + 35^2 - 2 \cdot 55 \cdot 35 \cdot \cos 42^\circ$$

$$b^2 = 1388.9 \dots$$

$$b = 37.3 \text{ mi}$$