Advanced Algebra
Chapter 4 Outline

4-1
10, 12, 14, 16, 18, 20, 22, 26, 27, 30, 31, 35, 36, 46, 47, 48, 50, 52, 54, 56 (20)

4-2
14, 16, 18, 20, 24, 26, 28, 30, 31, 32, 33, 34, 35, 42, 43, 45, 48, 50 (18)

4-3
14, 16, 18, 20, 22, 24, 26, 28, 31, 32, 33, 34, 40, 41, 42, 45, 47, 50 (18)

Worksheets 4-1, 4-2, 4-3

Quiz on 4-1, 4-2, 4-3

4-5
14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 49, 59, 66, 68, 70 (18)

4-7
10, 12, 14, 16, 17, 18, 19, 20, 22, 24, 26, 28, 32, 45, 46, 56, 59, 69, 76 (19)

4-8
12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 38, 42, 49 (15)

Review
pages 209-214, 2-48 even, skip 24, 26, 34, 36, 38
page 215, 19

Review
pages 209-214, 1-47 odd, skip 23, 25, 33, 35, 37

Chapter 4 Test

4.1 Introduction To Matrices

Objective: Solve data in matrices.
Solve equations involving matrices

Matrix: a rectangular array of variables or constants in horizontal rows and vertical columns.

Ex) $\begin{bmatrix} 2 & 4 \\ 7 & 6 \\ 9 & -3 \end{bmatrix}$

* Each #/value is an element
* Size: rows x columns ie: 3x2
* Square matrix
  - 3x3, 4x4, 5x5
* Zero matrix all entries are zero

Ex1) State the dimension of each

A) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

B) $\begin{bmatrix} 5 & 4 & 3 \\ -6 & 2 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$

Ex2) Solve each Equation

A) $\begin{bmatrix} x+4 \\ 2y \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \end{bmatrix}$

B) $\begin{bmatrix} y \\ 3x \end{bmatrix} = \begin{bmatrix} 6-2x \\ 31+4y \end{bmatrix}$

C) $\begin{bmatrix} 9 & 13 \end{bmatrix} = \begin{bmatrix} x+2y & 4x+1 \end{bmatrix}$

D) $\begin{bmatrix} 4x-3 & 3y \\ 7 & 13 \end{bmatrix} = \begin{bmatrix} 9 & -15 \\ 7 & 2z+1 \end{bmatrix}$

4-2 Operations of Matrices

Objective: Add and Subtract Matrices
Multiply by a Matrix by a scalar

Addition: A + B

Ex 1) $A = \begin{bmatrix} 6 & 4 \\ -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix}$

**Must be same size or can't add

$C = \begin{bmatrix} 4 & -2 & 0 \\ 1 & 5 & -1 \end{bmatrix}$ $D = \begin{bmatrix} -6 & 7 \\ -9 & 3 \end{bmatrix}$ C + D = DNE

Subtraction: A - B

Ex 2) $A = \begin{bmatrix} 9 & 2 \\ -4 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 6 \\ 8 & -2 \end{bmatrix}$

Scalar Multiplication: K[A]

Ex3) $A = \begin{bmatrix} 2 & 8 & -3 \\ 5 & -9 & 2 \end{bmatrix}$ Find 3A

Ex4) $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$ Find ½A

Properties For Addition

*A,B,C same dimensions and any scalar c, then

Commutative: A+B = B+A

Associative: (A+B)+C = A + (B+C)

Distributive: C(A+B) = CA = CB
"K" "K" "K"

Ex5) $A=\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ $B=\begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$ Find 4A - 3B

4.3 Multiplying Matrices

Objective: Multiply Matrices
Use the Properties of Matrix Multiplication

Multiply Matrices: You can multiply 2 matrices if and only if the # of columns in the 1st matrix is equal to the # of rows in the second matrix.

* These are the outer dimensions

$$A_{mxn} \text{ and } B_{nxr} = AB_{mxr}$$

*These have to be equal

Ex1) Determine whether each product is defined. If so, then state the product's dimensions.

A) $A_{2x5} + B_{5x4}$

B) $A_{3x2} + B_{4x3}$

C) $A_{3x4} + B_{4x2}$

Multiply Matrices

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \bullet \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$$

Ex2) Find RS if

$$R = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \text{ and } S = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$$

Ex3) Find KL and LK if

$$K = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix} \text{ and } L = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}$$

Properties for Multiplication

*Associative of Matrix Mult: (AB)C = A(BC)

*Associative for Scalar Mult: k(AB) = (kA)B = A(kB)

*Left Distributive: C(A+B) = CA + CB

*Right Distributive: (A+B)C = AC + BC

Matrix Multiplication is NOT communtative

4-5 Determinants

Objective: Evaluate the determinant of 2x2 & 3x3 matrix

A Determinant -is a square array of numbers or variables enclosed between two parallel lines.

To Evaluate a Second Order Determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

I. Find the value of each determinant

Ex1) $\begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix}$

Ex2) $\begin{vmatrix} -6 & 7 \\ -9 & 3 \end{vmatrix}$

To Evaluate a Third-Order Determinant

Way one: Expansion of minors

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Ex 3) $\begin{vmatrix} 2 & 7 & -3 \\ -1 & 5 & -4 \\ 6 & 9 & 0 \end{vmatrix}$

Ex 4) $\begin{vmatrix} 3 & -2 & -1 \\ 2 & -1 & 0 \\ 0 & -5 & 2 \end{vmatrix}$

Way two: Using Diagonals

Ex 5) $\begin{vmatrix} 2 & 7 & -3 \\ -1 & 5 & -4 \\ 6 & 9 & 0 \end{vmatrix}$

Ex 6) $\begin{vmatrix} 3 & -2 & -1 \\ 2 & -1 & 0 \\ 0 & -5 & 2 \end{vmatrix}$

Area of a Triangle

If (a, b), (c, d), and (e, f) are vertices, then

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Ex 7) Find the area of a triangle whose vertices are (0, -1), (-2, -6) and (3, -2).

4-7 Identity and Inverse Matrices

Objective: Determine whether 2 matrices are inverses.
Find the inverse of a 2 x 2.

Read page 195... $A \cdot A^{-1} = A^{-1} \cdot A = I$

Determine whether each pair of matrices are inverses.

Ex 1) $x = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ and $y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Ex 2) $P = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

Some matrices do not have an inverse. You can determine whether it does or does not by using the determinant...

ad - bc = det A (determinant of A)...If the determinant of A is 0, then the matrix does not have an inverse.

How to find an Inverse of a 2x 2.

$$A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix} \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse.

Ex 3) $R = \begin{bmatrix} -4 & -3 \\ 8 & 6 \end{bmatrix}$

Ex 4) $P = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Ex 5) $S = \begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix}$

4-8 Using Matrices to Solve Systems of Equations

Objective: Write matrix equations for systems.
Solve systems of equations using matrix equations.

Write a matrix equation for the system and solve.

Ex 1) $x + 3y = 3$
$x + 2y = 7$

Ex 2) $5x + 3y = 13$
$4x + 7y = -8$

Ex 3) $3x - 2y + z = 0$
$2x + 3y - z = 17$
$5x - y + 4z = -7$

AA – Study Guide for Chapter 4 – Matrices
Know these things below and you will be successful on your test.

4.1 Introduction to Matrices

- Write/Recognize Matrices from story problems
- Solve simple matrix equations like $\begin{bmatrix} x+3y \\ 3x+y \end{bmatrix} = \begin{bmatrix} -13 \\ 1 \end{bmatrix}$
- Determine a matrix's dimensions

4.2 Operations with Matrices

- Add/Subtract Matrices – Must be the same size
- Multiply a matrix by a scalar (constant)
- Combination of the above 2 statements
- Know properties of matrix operations (bottom of page 162)

4.3 Multiplying Matrices

- Multiply matrices – inner dimensions must be the same
- Know properties of matrix multiplication (top of page 171) and be able to show them as you did on the quiz.

~~4.4~~ skip

4.5 Determinants

- Find the determinant of a 2 x 2 matrix
- Find the determinant of a 3 x 3 matrix **TWO** ways:
  1. Expansion by minors
  2. Diagonals
- Find the area of a triangle when you are given just the vertices (page 184 – 185)

~~4.6~~ skip

4.7 Identity and Inverse Matrices

- Determine if 2 matrices are inverses (check A • B and B • A)
- Find the inverse of a 2 x 2 matrix (page 196)

4.8 Using Matrices to Solve Systems of Equations

- Write a matrix equation from a system of equations
- Solve systems with matrices (know what work you need to show)
- Set up systems of equations from word problems and then use matrices to solve

* Remember to show your work for all steps with matrices. You can ***check*** your answers with the graphing calculator. Solving systems with inverse multiplication (as you did in 4.8) is the only time you do not have to show all your work. Again, know what it is that you need to show for such problems.