Advanced Algebra
Chapter 5 Outline

5-1
18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 64, 66, 68, 70 (25)

5-2
16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 35, 36, 38, 40, 42, 44, 46, 48, 56, 57 (20)

5-3
16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 53, 54, 60, 61, 72 (20)

Worksheets 5-1 through 5-3

Quiz on 5-1 through 5-3

5-4
Day 1: worksheet
Day 2: worksheet
Day 3: worksheet

5-5
16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 58, 59, 60, 61, 66 (25)

5-6
Day 1: 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34 (20)
Day 2: 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 57, 58, 60, 62 (18)

Worksheets 5-4 through 5-6

Quiz 5-4 through 5-6

5-7
22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 67, 68 (20)

5-8
14, 16, 18, 20, 22, 24, 26, 28, 30, 39, 41, 45, 48, 50, 52 (15)

5-9
Day 1: 4, 5, 6, 7, 8, 18, 19, 20, 21, 22, 23, 24, 25 (13)
Day 2: 26, 28, 30, 32, 34, 36, 38, 40, 46, 48, 50, 52, 54, 56, 58, 62, 67, 68, 72, 74, 76 (20)

Review
Pages 277-280 12-74 evens, skip 24

Review
Pages 277-280, 11-75 odds

Chapter 5 Test

5-1 Monomials

Objective: Multiply and Divide Monomials
Use expressions written in scientific form.

Monomials:

$5b$, $-w$, 23, x^2, $1/3x^3y^3$

Not Monomials:

$1/n^4$, $\sqrt[3]{x}$, $x+8$, a^{-1}

-no variables in denominator

-no variables w/ negative exponent

-no variables under radicals

$5x^2y^4$

Coefficent:

Degree(sum of exp. variables):

Rules

1. **Negative Exponents**

$a^{-n}=1/a^n \qquad 1/a^{-n}=a^n$

Ex: $2^{-3}=1/2^3=1/8$

$\qquad 1/b^{-8}=b^{-8}$

2. **Product of Powers**

$a^m * a^n = a^{m+n}$

$4^2 * 4^6 = 4^8$

3. **Quotient of Powers**

$a^m/a^n = a^{m-n}$

$7^6/7^2 = 7^4$

Note: $5^0=1$, $6^0=1$, etc.

4. **Properties of Powers**

Power of a power: $(a^m)^n = a^{mn}$

$\qquad\qquad (7^2)^5 = 7^{10}$

Power of a product: $(ab)^n = a^n b^n$

$\qquad\qquad (5a)^2 = 25a^2$

Power of a quotient: $(a/b)^n = a^n/b^n$

$\qquad\qquad (x/y)^6 = x^6/y^6$

$\qquad\qquad (a/b)^{-n} = (b/a)^n = b^n/a^n$

$\qquad\qquad (x/y)^{-3} = y^3/x^3$

Simplify

Ex1: $(-2a^3b)(-5ab^4)$

Ex2: s^2/s^{10}

Ex 3: $(b^2)^4$

Ex 4: $(-3c^2d^5)^3$

Ex 5) $\left(\dfrac{-2a}{b^2}\right)^5$

Ex 6: $\left(\dfrac{x}{3}\right)^{-4}$

Ex 7: $\left(\dfrac{-3a^{5y}}{a^6y_b^4}\right)^5$

Scientific Notation: used to express very large or very small numbers

$$a \times 10^n \quad \text{where } 1 \le a < 10, \text{ n--any integer}$$

Ex 8: Express in Scientific Notation.

A) 4,560,000

B) .000092

Ex 9: Evaluate. Express each in Scientific Notation.

A) $(5 \times 10^3)(7 \times 10^8)$

B) $(1.8 \times 10^{-4})(4 \times 10^7)$

Ex 10: There are about 5×10^6 red blood cells in one ml of blood. A certain blood sample contains 8.32×10^6 red blood cells. About how many ml of blood are in the sample?

5-2 Polynomials

<u>**Objective:**</u> Add, subtract, and Multiply Polynomials.

<u>**Polynomial:**</u> a monomial or sum of monomials.

Binomial- 2 unlike terms   ie. $xy + z^3$

Trinomial- 3 unlike terms   ie. $x^2 + 3x + 1$

Degree of a Polynomial- The degree of the monomial with the greatest degree

$4x^3 + 3x^2$ degree 3 $5x^5y + 4x^3y^6$ degree 9

I. Determine whether each is a polynomial and state the degree.

A. $c^2 - 4\sqrt{c} + 18$

B. $-16p^5 + (3/4)p^2q^7$

II. Simplify

A. $(2a^3 + 5a - 7) - (a^3 - 3a + 2)$

B. $-y(4y^2 + 2y - 3)$

C. $(2p + 3)(4p + 1)$

D. $(a^2 + 3a - 4)(a + 2)$

E. $(n^2 + 6n - 2)(n + 4)$

5-3 Dividing Polynomials

Objective: Divide Polynomials using long and synthetic division.

I. Dividing by a monomial

Ex1) $\dfrac{5a^2b - 15ab^3 + 10a^3b^4}{5ab}$

II. Long Division
**Remember: 5 | 4612

EX 2. $(z^2 + 2z - 24)/(z - 4)$

EX 3. $(a^2 - 5a + 3)(a - 2)^{-1}$

EX 4. $(x^3 - 2x - 15)/(x - 5)$

III. Synthetic Division—is a faster way to divide polynomials. But, it can only be used when the divisor is in the form x – c. The coefficient in front of the divisor must be 1.

EX 5. $(x^3 - 4x^2 + 6x - 4)/(x - 2)$

EX 6. $(5x^4 - 13x^2 + 10x - 8)/(x + 1)$

5-4 Factoring of Polynomials Day 1

Objective: Factor polynomials.
Simplify polynomial quotients by factoring.

Greatest Common Factor (GCF)

EX 1. $10a^3b^2 + 15a^2b - 5ab^3$

EX 2. $6x^2y^2 - 2xy^2 + 6x^3y$

Difference of 2 Squares: $a^2 - b^2 = (a + b)(a - b)$

Sum of 2 Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of 2 Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Ex3) $x^2 - 36$

Ex4) $y^4 - z^2$

Ex5) $p^4 - 1$

Ex6) $2^3 + p^3$

Ex7) $c^3 d^3 + 27$

Ex8) $x^3 - y^3$

5 - 4 Day 2

Trinomials

Ex9) $x^2 + 7x + 6$

Ex10) $x^2 + 12x + 36$

Ex11) $x^2 - 6x - 27$

Ex12) $2a^2 + 3a + 1$

Ex13) $3n^2 + 21n - 24$

Ex14) $6c^2 + 13c + 6$

5-4 Day 3

Grouping

EX 1. $a^3 - 4a^2 + 3a - 12$

EX 2. $x^3 + 5x^2 - 2x - 10$

Simplifying Quotients

EX 3. $\dfrac{x^2 + 2x - 3}{x^2 + 7x + 12}$

EX 4. $\dfrac{a^2 - a - 6}{a^2 + 7a + 10}$

EX 5. $64x^6 - y^6$

5-5 Roots and Real Numbers

Objective: Simplify Radicals.
Use a calculator to approximate radicals.

Square Root: 7 is a square root of 49: 7x7=49.

6 is a square root of 36: 6x6=36.

-6 is a square root of 36: -6x-6=36.

6 (the nonnegative one) is called the **principal root**.

-Finding the square root and squaring are inverse operations.

-Raising a number to the nth power and finding the nth root are inverse operations.

Powers	**Factors**	**Roots**
$a^3 = 125$	5x5x5=125	5 is a cube root of 125
$a^4 = 81$	3x3x3x3=81	3 is a fourth root of 81
$a^5 = 32$	2x2x2x2x2=32	2 is a fifth root of 32
$a^n = b$	axa...a=b	a is an nth root of b.

$\sqrt[n]{50}$ indicates and nth root

n: index,   50: radicand, $\sqrt{}$: radical sign

-See chart on P.246.

Simplify

EX 1. $\pm\sqrt{16x^6}$

EX 2. $-\sqrt{(q^3+5)^4}$

EX 3. $\sqrt[5]{243a^{10}b^{15}}$

EX 4. $\sqrt{-4}$

EX 5. $\sqrt[8]{x^8}$

EX 6. $\sqrt[4]{81(a+1)^{12}}$

EX 7. $\sqrt[5]{243(x+2)^{15}}$

Approximate each value to the three decimal places.

EX 8. $\sqrt{4.27}$

EX 9. $-\sqrt{147}$

EX 10. $\sqrt[5]{891}$

EX 11. $\sqrt[4]{(3500)^2}$

5-6 Radical Expressions Day 1

Objective: Simplify radical expressions.
Add, Subtract, multiply, and divide radical expressions.

Product Property of Radicals n>1
1. If n is even and a and b are nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

2. If n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Steps to Simplifying a square root
1. Factor the radicand into as many squares as possible
2. Use the product property to isolate the perfect squares
3. Simplify each

$$\sqrt{\text{Perfect Square Factors}} \cdot \sqrt{\text{Other Factors}}$$

Simplify

Ex1) $\sqrt{16p^8q^7} =$

Ex2) $\sqrt{25a^4b^9} =$

Quotient Property

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Simplifying Quotients

Ex3) $\sqrt{\dfrac{y^8}{x^7}}$

Ex4) $\sqrt[3]{\dfrac{2}{9x}}$

Ex5) $\sqrt[5]{\dfrac{5}{4a}}$

Multiplying Radicals

Ex6) $6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n}$

Ex7) $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$

"Mixed" Simplifying

Ex8) $\sqrt[4]{16x^5y^4}$

Ex9) $\sqrt{\dfrac{7}{8y}}$

Ex10) $-2\sqrt{15} \cdot 4\sqrt{21}$

5.6 Radical Expressions Day 2
** Bottom of Page 252 **

Add and Subtract Radicals

Ex1) $2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$ Ex2) $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$

Multiplying Radials

Ex3) $\left(3\sqrt{5} - 2\sqrt{3}\right)\left(2 - \sqrt{3}\right)$ Ex4) $\left(4\sqrt{2} + 7\right)\left(4\sqrt{2} - 7\right)$

Rationalizing Denominator using Conjugates:

$a\sqrt{b} - c\sqrt{d}$ and $a\sqrt{b} + c\sqrt{d}$ are conjugates.

EX 5. $\dfrac{1 - \sqrt{3}}{5 + \sqrt{3}}$

EX 6. $\dfrac{2 + \sqrt{3}}{4 - \sqrt{3}}$

<u>5-7 Rational Exponents</u>

<u>Objective</u>:   Write expressions with rational exponents in radical
from and vice versa.
Simplify expressions in exponential or radical form.

Key Ideas

$b^{1/n} = \sqrt[n]{b}$

$b^{m/n} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$

$b^{m/n} = (b^m)^{1/n} = (b^{1/n})^m$

<u>I. Radical Form</u> - Write each expression in
radical form.

EX 1. $a^{1/4}$

EX 2. $a^{5/6}$

EX 3. $m^{3/2}$

<u>II. Exponential Form</u> - Write each radical using rational exponents.

EX 4. $\sqrt[5]{b}$

EX 5. $\sqrt{w^5}$

EX 6. $\sqrt[8]{c^3}$

III. Evaluate

EX 7. $49^{-1/2}$

EX 8. $32^{2/5}$

EX 9. $243^{3/5}$

IV. Simplify
-Read P.260
-If you start the problem with radicals, answer with radicals. If you start with rational exponents, answer with rational exponents.

EX 10. $x^{1/5} \cdot x^{7/5}$

EX 11. $y^{-3/4}$

-To reduce the index, look for perfect squares, cubes, 4ths, then rewrite and simplify.
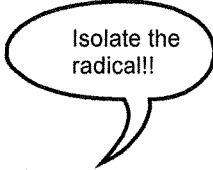
EX 12. $\dfrac{\sqrt[8]{81}}{\sqrt[6]{3}}$

EX 13. $\sqrt[6]{4x^4}$

EX 14. $\sqrt[4]{9z^2}$

5-8 Radical Equations and Inequalities

Isolate the radical!!

Objective: Solve equations and inequalities containing radicals.

I. Radical Equations

EX 1. $\sqrt{(x+1)} + 2 = 4$

EX 2. $\sqrt{(y-2)} - 1 = 5$

EX 3. $\sqrt{x - 15} = 3 - \sqrt{x}$

II. Cube Root Equations

EX 4. $3(5n-1)^{1/3} - 2 = 0$

EX 5. $(3y+1)^{1/3} + 5 = 0$

III. Radical Inequalities

EX 6. $2 + \sqrt{(4x - 4)} \le 6$

EX 7. $\sqrt{(3x - 6)} + 4 \le 7$

<u>5-9 Complex Numbers</u>
<u>Objective</u>: Add, subtract, multiply and divide complex numbers.
<u>Day 1</u>
Where did the imaginary number come from????

<u>So let us take a look at what happens to the i.</u>

<u>Summary of the things you need to know for today:</u>
<u>Standard Form</u>: $a + bi$, $i^2 = -1$, $i = \sqrt{-1}$

— —

<u>II. Simplify</u>.

EX 1. $\sqrt{-18}$

EX 2. $\sqrt{-28}$

EX 3. $\sqrt{-125x^5}$

EX 4. $\sqrt{-32y^3}$

EX 5. $-2i \cdot 7i$

EX 6. $\sqrt{-12} \cdot \sqrt{-2}$

Day 2 on 5-9
Simplify.

EX 7. i^{35}

EX 8. $(6 - 4i) + (1 + 3i)$

EX 9. $(4 - 6i) - (1 + 3i)$

EX 10. $\dfrac{5i}{3 + 2i}$

EX 11. $\dfrac{4 - i}{5i}$

Solve the equation.

EX 12. $3x^2 + 48 = 0$ EX 13. $5y^2 + 20 = 0$

AA Ch 5 Study Guide

5.1   Monomials
- Simplify monomials (1-term polynomials)
- No negative exponents allowed
- Know what an exponent of zero means
- Know rules for when to add, subtract, multiply exponents
- Divide Monomials
- Know scientific notation – WORD PROBLEM with scientific notation

5.2   Polynomials
- Add & subtract polynomials – combine like terms
- Multiply polynomials (distributive property/FOIL)

5.3   Dividing Polynomials
- Long Division & Synthetic Division (MUST know both!)
- Remember to fill missing terms with zero(s)

5.4   Factoring Polynomials
- Factor out a Monomial
- Factor Difference of Squares
- Factor Difference of Cubes
- Factor Sum of Cubes
- Factor Trinomials $ax^2 + bx + c$ with $a = 1$
- Factor Trinomials $ax^2 + bx + c$ with a not 1
- Factor with Grouping
- Factor with Division problems

5.5   Roots of Real Numbers
- Know when a negative in the radicand is possible
- Find nth roots of perfect squares, perfect cubes, perfect fourths, etc.

5.6   Radical Expressions
- Simplify roots when they are NOT perfect
- Multiply with roots and simplify
- Add/Subtract roots (remember only if the radicands are the same)
- FOIL (multiply and add/subtract) with roots
- Rationalize denominator (when denominator is a monomial)
- Rationalize denominator (when denominator is a binomial) – Multiply by its conjugate

5.7   Rational Exponents
- Interchange between radical form and using rational exponents
- Simplify – remember no negative exponents and no fractional exponents in denominator (the only way you get good at this is with lots of exposure to a variety of problems – which means practice!)

5.8   Radical Equations and Inequalities
- Solve equations/inequalities with radicals, remembering to ALWAYS check your answers
- WORD PROBLEM with solving radical equations

5.9   Complex Numbers
- Simplify radicals of negative numbers
- Add/Subtract/Multiply/Divide with complex numbers
- No i allowed in denominator – multiply by complex conjugate
- WORD PROBLEM with complex numbers like in homework and on page 273