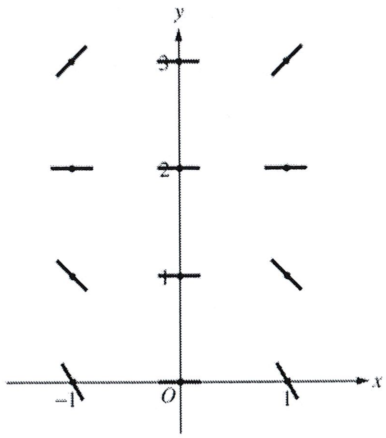
Slope Field Solutions

(1)

(a)



2 : { 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$
1 : { positive slope at each point (x, y) where $x \neq 0$ and $y > 2$
negative slope at each point (x, y) where $x \neq 0$ and $y < 2$ } }

(b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

1 : description

(c)
$$\frac{1}{y-2}dy = x^4 dx$$
$$\ln|y-2| = \frac{1}{5}x^5 + C$$
$$|y-2| = e^C e^{\frac{1}{5}x^5}$$
$$y - 2 = Ke^{\frac{1}{5}x^5}, \quad K = \pm e^C$$
$$-2 = Ke^0 = K$$
$$y = 2 - 2e^{\frac{1}{5}x^5}$$

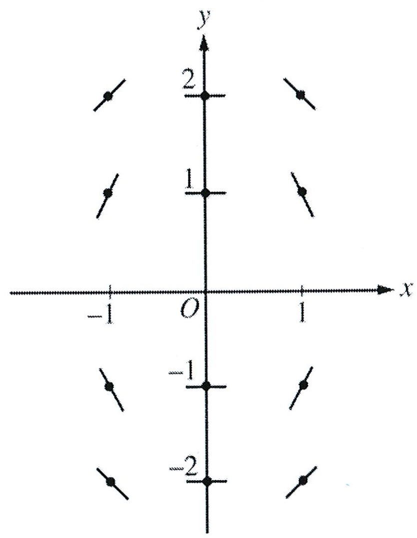
6 : {
1 : separates variables
2 : antiderivatives
1 : constant of integration
1 : uses initial condition
1 : solves for y
0/1 if y is not exponential }

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

2

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) The line tangent to f at $(1, -1)$ is $y + 1 = 2(x - 1)$.
Thus, $f(1.1)$ is approximately -0.8.

$2 : \begin{cases} 1 : \text{equation of the tangent line} \\ 1 : \text{approximation for } f(1.1) \end{cases}$

(c) $\dfrac{dy}{dx} = -\dfrac{2x}{y}$

$y\,dy = -2x\,dx$

$\dfrac{y^2}{2} = -x^2 + C$

$\dfrac{1}{2} = -1 + C;\ C = \dfrac{3}{2}$

$y^2 = -2x^2 + 3$

Since the particular solution goes through $(1, -1)$, y must be negative.

Thus the particular solution is $y = -\sqrt{3 - 2x^2}$.

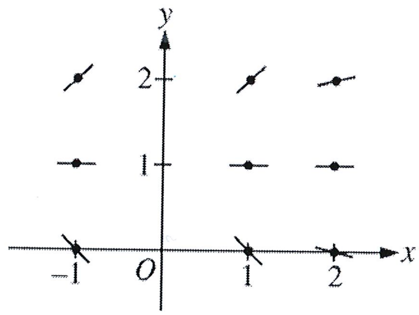
$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

(3)

(a)



$2: \begin{cases} 1: \text{zero slopes} \\ 1: \text{all other slopes} \end{cases}$

(b) $\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^{C}e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^{C}$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)}$, $x > 0$

$6: \begin{cases} 1: \text{separates variables} \\ 2: \text{antidifferentiates} \\ 1: \text{includes constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(c) $\lim_{x\to\infty} 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit

Area and Volume

1

(a) Area $= \int_1^{10} \sqrt{x-1}\, dx = 18$	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_1^{10} \left(9 - \left(3 - \sqrt{x-1}\right)^2\right) dx$ $= 212.057$ or 212.058	$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(c) Volume $= \pi \int_0^3 \left(10 - \left(y^2 + 1\right)\right)^2 dy$ $= 407.150$	$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

2

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$
(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx -$ Area of R

$= 2.06016 -$ Area of $R = 1.546$

OR

$$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{constant and limits} \end{cases}$

3

Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

1 : Correct limits in an integral in (a), (b), or (c).

(a) Area $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$

$= 0.590$ or 0.591

2 { 1 : integrand
1 : answer }

(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right) dx$

$= 0.55596\pi = 1.746$ or 1.747

3 { 2 : integrand and constant
< −1> each error
1 : answer }

(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

3 { 2 : integrand
< −1> each error
Note: 0/2 if not of the form

$$k \int_c^d (f(x) - g(x))^2 \, dx$$

1 : answer }

Riemann Sums

1

(a) $\int_{30}^{60} |v(t)|\,dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\,dt$:

$$A = \frac{1}{2}(14+10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

$2 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

(b) $\int_{0}^{30} a(t)\,dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\int_{0}^{30} a(t)\,dt = \int_{0}^{30} v'(t)\,dt = v(30) - v(0)$$
$$= -14 - (-20) = 6 \text{ ft/sec}$$

$2 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

$2 : \begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

$2 : \begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$

Units of ft in (a) and ft/sec in (b)

1 : units in (a) and (b)

2

(a) $\frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{2} = -\frac{7}{2}\,°\text{C/cm}$

1 : answer

(b) $\frac{1}{8}\int_{0}^{8} T(x)\,dx$

Trapezoidal approximation for $\int_{0}^{8} T(x)\,dx$:

$$A = \frac{100+93}{2}\cdot 1 + \frac{93+70}{2}\cdot 4 + \frac{70+62}{2}\cdot 1 + \frac{62+55}{2}\cdot 2$$

Average temperature $\approx \frac{1}{8}A = 75.6875°\text{C}$

$3 : \begin{cases} 1 : \frac{1}{8}\int_{0}^{8} T(x)\,dx \\ 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

(c) $\int_{0}^{8} T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45°\text{C}$

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$

(d) Average rate of change of temperature on $[1, 5]$ is $\frac{70 - 93}{5 - 1} = -5.75$.

Average rate of change of temperature on $[5, 6]$ is $\frac{62 - 70}{6 - 5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in $[0, 8]$.

$2 : \begin{cases} 1 : \text{two slopes of secant lines} \\ 1 : \text{answer with explanation} \end{cases}$

Units of °C/cm in (a), and °C in (b) and (c)

1 : units in (a), (b), and (c)

③

(a) Average acceleration of rocket A is $\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} = \frac{11}{20}$ ft/sec^2	1 : answer
(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is $20[v(20)+v(40)+v(60)]$ $= 20[22+35+44] = 2020$ ft	3 : { 1 : explanation 1 : uses $v(20)$, $v(40)$, $v(60)$ 1 : value
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$ $2 = v_B(0) = 6 + C$ $v_B(t) = 6\sqrt{t+1} - 4$ $v_B(80) = 50 > 49 = v(80)$ Rocket B is traveling faster at time $t = 80$ seconds.	4 : { 1 : $6\sqrt{t+1}$ 1 : constant of integration 1 : uses initial condition 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion
Units of ft/sec^2 in (a) and ft in (b)	1 : units in (a) and (b)

Rates

1

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

2

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000$ cm^3/min and $\frac{dr}{dt} = 2.5$ cm/min. $\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$ $2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$ $\frac{dh}{dt} = 0.038$ or 0.039 cm/min	$4: \begin{cases} 1: \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2: \text{expression for } \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$. This occurs when $t = 25$ minutes. Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.	$3: \begin{cases} 1: R(t) = 2000 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\,dt$.	$2: \begin{cases} 1: \text{limits and initial condition} \\ 1: \text{integrand} \end{cases}$

(3)

(a) $\int_9^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

3:
- 1 : limits
- 1 : integrand
- 1 : answer

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was $104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was

$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$

1 : setup

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

3:
- 1 : value of $H'(17)$
- 2 : meanings
  - 1 : meaning of $H(17)$
  - 1 : meaning of $H'(17)$
  - $< -1 >$ if no reference to $t = 17$

(d) $H'(t) = E(t) - L(t) = 0$

$t = 15.794$ or 15.795

2:
- 1 : $E(t) - L(t) = 0$
- 1 : answer

IMPLICIT DIFFERENTIAN

(1)

(a) $y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

$2\begin{cases}1: \text{implicit differentiation}\\ 1: \text{verifies expression for } \frac{dy}{dx}\end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3,\ y = -2$

At $(1,3)$, $\frac{dy}{dx} = \frac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\frac{dy}{dx} = \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

$4\begin{cases}1: y^2 - y = 6\\ 1: \text{solves for } y\\ 2: \text{tangent lines}\end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x(2y - x^2) = 0$ gives $x = 0$ or $y = \frac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \frac{1}{2}x^2$, $\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$-\frac{1}{4}x^5 = 6$

$x = \sqrt[5]{-24}$

$3\begin{cases}1: \text{sets denominator of } \frac{dy}{dx} \text{ equal to } 0\\ 1: \text{substitutes } y = \frac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \quad \text{into the equation for the curve}\\ 1: \text{solves for } x\text{-coordinate}\end{cases}$

(2)

(a) $\dfrac{d^2y}{dx^2} = 2y\dfrac{dy}{dx}(6-2x) - 2y^2$

$= 2y^3(6-2x)^2 - 2y^2$

$\left.\dfrac{d^2y}{dx^2}\right|_{\left(3,\frac{1}{4}\right)} = 0 - 2\left(\dfrac{1}{4}\right)^2 = -\dfrac{1}{8}$

$3: \begin{cases} 2: \dfrac{d^2y}{dx^2} \\ \quad <-2> \text{ product rule or chain rule error} \\ 1: \text{value at } \left(3, \dfrac{1}{4}\right) \end{cases}$

(b) $\dfrac{1}{y^2}\,dy = (6-2x)\,dx$

$-\dfrac{1}{y} = 6x - x^2 + C$

$-4 = 18 - 9 + C = 9 + C$

$C = -13$

$y = \dfrac{1}{x^2 - 6x + 13}$

$6: \begin{cases} 1: \text{separates variables} \\ 1: \text{antiderivative of } dy \text{ term} \\ 1: \text{antiderivative of } dx \text{ term} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition } f(3) = \dfrac{1}{4} \\ 1: \text{solves for } y \end{cases}$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(3)

(a) $2yy' = y + xy'$

$(2y - x)y' = y$

$y' = \dfrac{y}{2y-x}$

$2: \begin{cases} 1: \text{implicit differentiation} \\ 1: \text{solves for } y' \end{cases}$

(b) $\dfrac{y}{2y-x} = \dfrac{1}{2}$

$2y = 2y - x$

$x = 0$

$y = \pm\sqrt{2}$

$(0, \sqrt{2}), (0, -\sqrt{2})$

$2: \begin{cases} 1: \dfrac{y}{2y-x} = \dfrac{1}{2} \\ 1: \text{answer} \end{cases}$

(c) $\dfrac{y}{2y-x} = 0$

$y = 0$

The curve has no horizontal tangent since $0^2 \neq 2 + x \cdot 0$ for any x.

$2: \begin{cases} 1: y = 0 \\ 1: \text{explanation} \end{cases}$

(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \dfrac{7}{3}$.

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt} = \dfrac{y}{2y-x} \cdot \dfrac{dx}{dt}$

At $t = 5$, $6 = \dfrac{3}{6 - \frac{7}{3}} \cdot \dfrac{dx}{dt} = \dfrac{9}{11} \cdot \dfrac{dx}{dt}$

$\left.\dfrac{dx}{dt}\right|_{t=5} = \dfrac{22}{3}$

$3: \begin{cases} 1: \text{solves for } x \\ 1: \text{chain rule} \\ 1: \text{answer} \end{cases}$

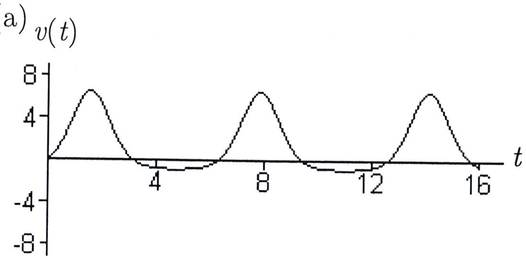
Particle in Motion

(1)

(a) $v(1.5) = 1.5\sin(1.5^2) = 1.167$ Up, because $v(1.5) > 0$	**1**: answer and reason
(b) $a(t) = v'(t) = \sin t^2 + 2t^2\cos t^2$ $a(1.5) = v'(1.5) = -2.048$ or -2.049 No; v is decreasing at 1.5 because $v'(1.5) < 0$	**2** $\begin{cases} 1\text{: } a(1.5) \\ 1\text{: conclusion and reason} \end{cases}$
(c) $y(t) = \int v(t)\,dt$ $= \int t\sin t^2\,dt = -\frac{\cos t^2}{2} + C$ $y(0) = 3 = -\frac{1}{2} + C \implies C = \frac{7}{2}$ $y(t) = -\frac{1}{2}\cos t^2 + \frac{7}{2}$ $y(2) = -\frac{1}{2}\cos 4 + \frac{7}{2} = 3.826$ or 3.827	**3** $\begin{cases} 1\text{: } y(t) = \int v(t)\,dt \\ 1\text{: } y(t) = -\frac{1}{2}\cos t^2 + C \\ 1\text{: } y(2) \end{cases}$
(d) distance $= \int_0^2 \lvert v(t)\rvert\,dt = 1.173$ or $v(t) = t\sin t^2 = 0$ $t = 0$ or $t = \sqrt{\pi} \approx 1.772$ $y(0) = 3$; $y(\sqrt{\pi}) = 4$; $y(2) = 3.826$ or 3.827 $[y(\sqrt{\pi}) - y(0)] + [y(\sqrt{\pi}) - y(2)]$ $= 1.173$ or 1.174	**3** 1: limits of 0 and 2 on an integral of $v(t)$ or $\lvert v(t)\rvert$ or uses $y(0)$ and $y(2)$ to compute distance 1: handles change of direction at student's turning point 1: answer 0/1 if incorrect turning point

2

(a)



1 : graph
three "humps"
periodic behavior
starts at origin
reasonable relative max and min values

(b) Particle is moving to the left when $v(t) < 0$, i.e. $e^{2\sin t} < 1$.

$(\pi, 2\pi)$, $(3\pi, 4\pi)$ and $(5\pi, 16]$

3 { 2 : intervals
< −1 > each missing or incorrect interval
1 : reason }

(c) $\int_0^4 |v(t)|\,dt = 10.542$

or

$v(t) = e^{2\sin t} - 1 = 0$

$t = 0$ or $t = \pi$

$x(\pi) = \int_0^\pi v(t)\,dt = 10.10656$

$x(4) = \int_0^4 v(t)\,dt = 9.67066$

$|x(\pi) - x(0)| + |x(4) - x(\pi)|$

$= 10.542$

3 { 1 : limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$
or
uses $x(0)$ and $x(4)$ to compute distance
1 : handles change of direction at student's turning point
1 : answer
note: 0/1 if incorrect turning point }

(d) There is no such time because $\int_0^T v(t)\,dt > 0$ for all $T > 0$.

2 { 1 : no such time
1 : reason }

3

(a) $a(4) = v'(4) = \frac{5}{7}$	1 : answer
(b) $v(t) = 0$ $t^2 - 3t + 3 = 1$ $t^2 - 3t + 2 = 0$ $(t-2)(t-1) = 0$ $t = 1, 2$ $v(t) > 0$ for $0 < t < 1$ $v(t) < 0$ for $1 < t < 2$ $v(t) > 0$ for $2 < t < 5$ The particle changes direction when $t = 1$ and $t = 2$. The particle travels to the left when $1 < t < 2$.	3 : { 1 : sets $v(t) = 0$ 1 : direction change at $t = 1, 2$ 1 : interval with reason
(c) $s(t) = s(0) + \int_0^t \ln\left(u^2 - 3u + 3\right) du$ $s(2) = 8 + \int_0^2 \ln\left(u^2 - 3u + 3\right) du$ $= 8.368$ or 8.369	3 : { 1 : $\int_0^2 \ln\left(u^2 - 3u + 3\right) du$ 1 : handles initial condition 1 : answer
(d) $\frac{1}{2}\int_0^2 \lvert v(t)\rvert\, dt = 0.370$ or 0.371	2 : { 1 : integral 1 : answer

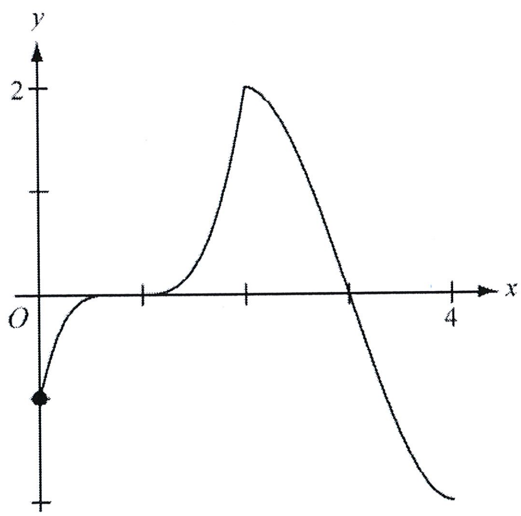
f, f', f'' Graphs

(1)

(a) f has a relative maximum at $x = 2$ because f' changes from positive to negative at $x = 2$.

$2: \begin{cases} 1: \text{relative extremum at } x = 2 \\ 1: \text{relative maximum with justification} \end{cases}$

(b)



$2: \begin{cases} 1: \text{points at } x = 0, 1, 2, 3 \\ \quad \text{and behavior at } (2, 2) \\ 1: \text{appropriate increasing/decreasing} \\ \quad \text{and concavity behavior} \end{cases}$

(c) $g'(x) = f(x) = 0$ at $x = 1, 3$.
g' changes from negative to positive at $x = 1$ so g has a relative minimum at $x = 1$. g' changes from positive to negative at $x = 3$ so g has a relative maximum at $x = 3$.

$3: \begin{cases} 1: g'(x) = f(x) \\ 1: \text{critical points} \\ 1: \text{answer with justification} \end{cases}$

(d) The graph of g has a point of inflection at $x = 2$ because $g'' = f'$ changes sign at $x = 2$.

$2: \begin{cases} 1: x = 2 \\ 1: \text{answer with justification} \end{cases}$

(2)

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.

$2: \begin{cases} 1: h(1) \text{ and } h(3) \\ 1: \text{conclusion, using IVT} \end{cases}$

(b) $\dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{3 - 1} = -5$
Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c, $1 < c < 3$, such that $h'(c) = -5$.

$2: \begin{cases} 1: \dfrac{h(3) - h(1)}{3 - 1} \\ 1: \text{conclusion, using MVT} \end{cases}$

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

$2: \begin{cases} 1: \text{apply chain rule} \\ 1: \text{answer} \end{cases}$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.

$$\left(g^{-1}\right)'(2) = \frac{1}{g'\left(g^{-1}(2)\right)} = \frac{1}{g'(1)} = \frac{1}{5}$$

An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.

$3: \begin{cases} 1: g^{-1}(2) \\ 1: \left(g^{-1}\right)'(2) \\ 1: \text{tangent line equation} \end{cases}$

3

(a) $\int_0^{1.5} (3f'(x)+4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$ $= 3f(x) + 4x\Big\vert_0^{1.5} = 3(-1-(-7)) + 4(1.5) = 24$	$2 \begin{cases} 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$
(b) $y = 5(x-1) - 4$ $f(1.2) \approx 5(0.2) - 4 = -3$ The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.	$3 \begin{cases} 1: \text{tangent line} \\ 1: \text{computes } y \text{ on tangent line at } x = 1.2 \\ 1: \text{answer with reason} \end{cases}$
(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that $f''(c) = \frac{f'(0.5) - f'(0)}{0.5 - 0} = \frac{3-0}{0.5} = 6 = r$	$2 \begin{cases} 1: \text{reference to MVT for } f' \text{ (or differentiability of } f'\text{)} \\ 1: \text{value of } r \text{ for interval } 0 \le x \le 0.5 \end{cases}$
(d) $\lim_{x\to 0^-} g'(x) = \lim_{x\to 0^-} (4x-1) = -1$ $\lim_{x\to 0^+} g'(x) = \lim_{x\to 0^+} (4x+1) = +1$ Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$. OR $g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.	$2 \begin{cases} 1: \text{answers “no” with reference to } g' \text{ or } g'' \\ 1: \text{correct reason} \end{cases}$