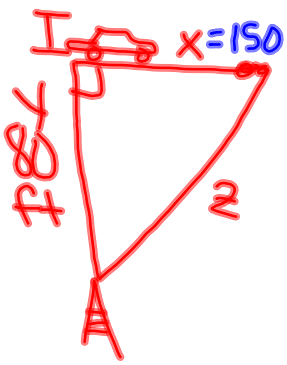
4.6 Related Rates Day 2

Ex 1) A car is headed east from an intersection at 50 ft/sec. A radio tower is 80 ft south of the intersection. How fast is the distance between the car and the radio station changing at t = 3 seconds?

$\frac{dx}{dt} = 50\text{ft/sec}$ $\quad \frac{dz}{dt} = ?$ $\quad \frac{dy}{dt} = 0$

$t = 3$



$x^2 + y^2 = z^2$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$

$X = 50 \cdot 3 = 150$

$2 \cdot 150 \cdot 50 + \cancel{2 \cdot 80} \cdot 0 = 2 \cdot 170 \cdot \frac{dz}{dt}$

$80^2 + 150^2 = z^2$

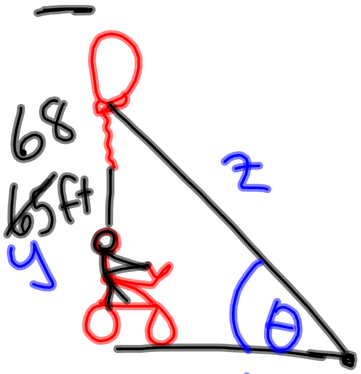
$170 = z$

$15{,}000 = 340 \frac{dz}{dt}$

$\frac{15000}{340} = \frac{dz}{dt}$

$\boxed{44.11\text{ ft/sec} = \frac{dz}{dt}}$

Ex 2) A balloon rises 1 ft/sec. When a bike is traveling at 17 ft/sec is directly below the balloon, the balloon's height is 65 feet. How fast is the angle of elevation changing at t = 3 seconds?



$x = 51$

$\frac{dy}{dt} = 1$ ft/sec

$t = 3$

$\frac{d\theta}{dt} = ?$

$\frac{dx}{dt} = 17$ ft/sec

$\tan\theta = \frac{y}{x}$

$x = 3 \cdot 17 = 51$

$y = 65 + 3 = 68$

$\tan\theta = 68/51$

$\theta = 53.13$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt}}{x^2}$$

$$\sec^2(53.13) \cdot \frac{d\theta}{dt} = \frac{(51 \cdot 1 - 68 \cdot 17)}{(51^2)}$$

$$\frac{d\theta}{dt} = \frac{-.425}{\sec^2(53.13)}$$

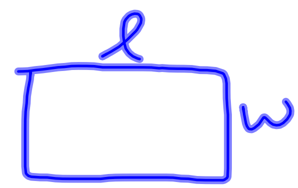
$$= -.425 \times \cos^2(53.13)$$

$$= \boxed{-.153 \text{ rad/sec}}$$

Ex 3) The length of a rectangle increases at 2 in/min and the width decreases at 2 in/min. What happens to the area?

$\frac{dw}{dt} = -2$

$\frac{dl}{dt}$



~~A~~. Area always increases

~~B~~. Area always decreases

~~C~~. Area is always constant

D. Area increases when w>l

E. Area increases when l>w

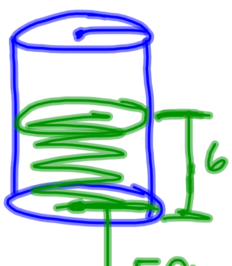
$A = l \cdot w$

$\frac{dA}{dt} = l \cdot \frac{dw}{dt} + w \cdot \frac{dl}{dt}$

$\frac{dA}{dt} = l \cdot -2 + w \cdot 2$

$\frac{dA}{dt} = +$

Ex 4) Water runs into a cylindrical tank at 9 ft³/min. How fast is the water level rising when h=6ft and r = 5ft?



$\frac{dV}{dt} = 9\,ft^3/min$ $\qquad$ $\frac{dh}{dt} = ?$

$\frac{dr}{dt} = 0$

$V = \pi r^2 h$

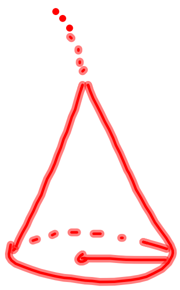
$\frac{dV}{dt} = h \cdot 2\pi r \cdot \frac{dr}{dt} + \pi r^2 \cdot \frac{dh}{dt}$

$9 = 6 \cdot 2\pi \cdot 5 \cdot 0 + \pi (5)^2 \cdot \frac{dh}{dt}$

$9 = 25\pi \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{(25\pi)} = .114\,ft/min$

Ex 5) Wheat is pouring from a chute at 10 ft³/min and forms a conical pile. The radius is half the height. How fast is the height rising when the pile is 8 feet high?



$\frac{dh}{dt} = ?$ $\quad \frac{dV}{dt} = +10 ft^3/min$ $\quad r = \frac{1}{2}h$

$h = 8 ft$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$$

$$V = \frac{\pi h^3}{12}$$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{12}} \cdot \cancel{3}h^2 \cdot \frac{dh}{dt}$$

$$10 = \frac{\pi}{4}(8)^2 \cdot \frac{dh}{dt}$$

$$10 = 16\pi \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{10}{16\pi} = .199$$

$$= \boxed{.199 ft/min}$$