2.1 - Rates of Change and Limits

Avg. speed = $\dfrac{\text{Distance traveled}}{\text{time elapsed}}$

$S = \text{position}$

Ex 1) A ball is thrown. Its height is modeled by $h(t) = 40t - 16t^2$

Find the avgerage speed between $t = 0$ and $t = 1$

$$\text{AvgSpeed} = \frac{\Delta S}{\Delta T} = \frac{h(1)-h(0)}{1-0} = \frac{24-0}{1} = \boxed{24\ m/s}$$

$$h(1) = 40(1) - 16(1)^2 = 40 - 16 = 24$$

$$h(0) = 40(0) - 16(0)^2 = 0$$

Instantaneous Speed = Speed at a given point (derivative)

Ex 2) Find the instantaneous speed at t = 1 of $h(t)=40t-16t^2$

Choose t = 1 and t = 1+x, with x infinitely close to 0

$$\frac{\Delta S}{\Delta t} = \frac{h(1+x) - h(1)}{1+x-1}$$

$$= \frac{40(1+x) - 16(1+x)^2 - (40(1) - 16(1)^2)}{x}$$

$$= \frac{\cancel{40} + 40x \cancel{-16} - 16x^2 - 32x \cancel{-40} \cancel{+16}}{x}$$

$$= \frac{8x - 16x^2}{x} = \frac{\cancel{x}(8-16x)}{\cancel{x}} = 8 - 16x$$

$$\lim_{x \to 0} 8 - 16x = \boxed{8\ m/sec}$$

Limits

$\lim_{x \to c} f(x) = L$ read "the limit of $f(x)$ as x approaches c is L"

The values of $f(x)$ approach or equal the value of L as the values of x approach (but are not necessarily equal to) c).

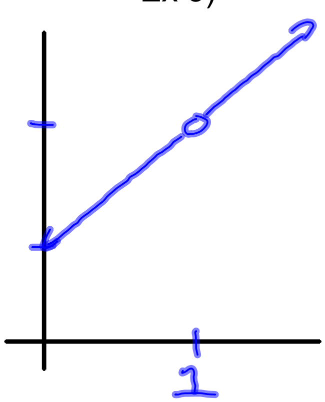
So you're really asking yourself...

What does y get close to as x gets close to some number c?

3 methods to evaluate limits

1. Graphically
2. Numerically (using the table)
3. Algebraically (no calculators)

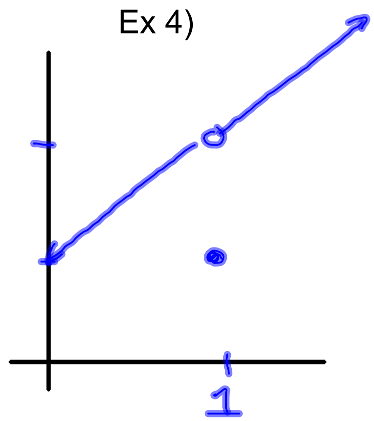
Ex 3)



$\lim_{x \to 1} f(x) = 2$

$f(1) = \text{DNE}$

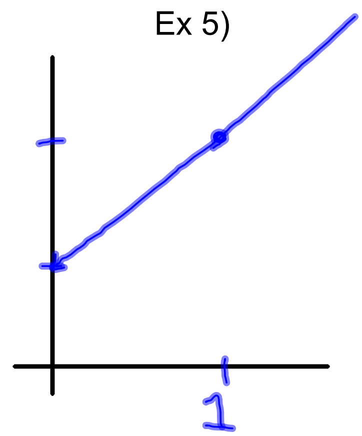
Ex 4)



$\lim_{x \to 1} f(x) = 2$

$f(1) = 1$

Ex 5)

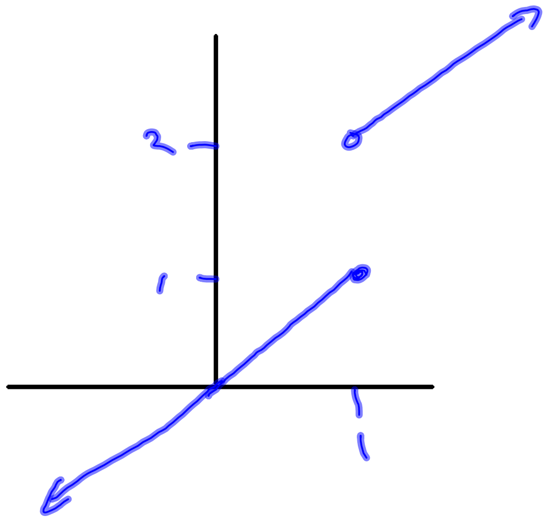


$\lim_{x \to 1} f(x) = 2$

$f(1) = 2$

The existence of a limit as $x \to c$ never depends on whether $f(x)$ is defined at c.

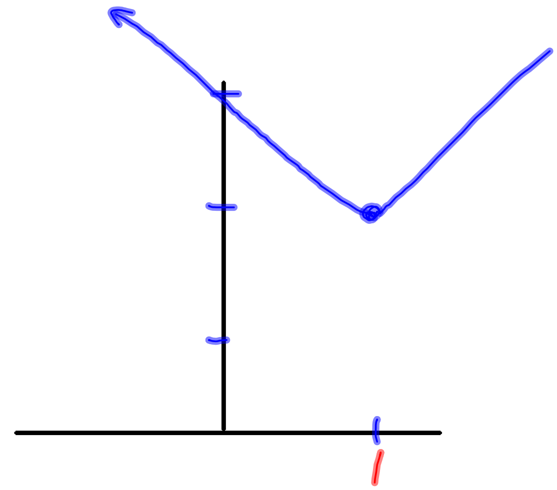
Ex 6)



$\lim_{x\to 1} f(x) =$ DNE

$f(1)=1$

Ex 7)



$\lim_{x\to 1} f(x) = 2$

$f(1)=2$

You can also find the limit by **Substitution**

If f(x) is a polynomial function, then

$$\lim_{x\to c} f(x) = f(c)$$

If f(x) is a rational function, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$$

Ex 8) $\lim_{x \to 4} 5x^2 - 2x + 3 = 5(4)^2 - 2(4) + 3$

$= 80 - 8 + 3 = 75$

Ex 9) $\lim_{x \to -1} \frac{x - 2}{x^2 + 4x - 3} = \frac{-1-2}{(-1)^2 + 4(-1) - 3}$

$= \frac{-3}{-6} = \frac{1}{2}$