2.1 Rates of Change and Limits Day 2

Let us take a look at what happens to this function when using substitution to find the limit.

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{0}{0}$$

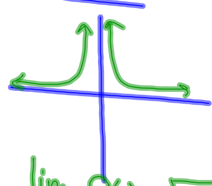
we can't determine anything (indeterminate form)

But, when $\lim_{x\to c} f(x) = \frac{0}{\#} = 0$

when $\lim_{x\to c} f(x) = \frac{\#}{0}$

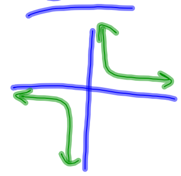
2 possibilities → $\pm\infty$ → DNE

$\pm\infty$



$\lim_{x\to 0} f(x) = \boxed{\infty}$

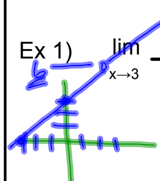
DNE



$\lim_{x\to 0} f(x) = \text{DNE}$

So, let us try finding the limit another way using factoring.

Ex 1) $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x\to 3} x+3$



$= 3+3 = \boxed{6}$

Ex 2) $\lim_{x\to 1} \frac{t^3-t}{t-3} = \frac{1^3-1}{1-3} = \frac{0}{-2} = \boxed{0}$

Ex 3) $\lim_{h\to 0} \frac{(h-5)^2-25}{h} = \lim_{h\to 0} \frac{h^2-10h+\cancel{25}-\cancel{25}}{h}$

$= \lim_{h\to 0} \frac{h^2-10h}{h} = \lim_{h\to 0} \frac{\cancel{h}(h-10)}{\cancel{h}} = \lim_{h\to 0} h-10$

$= 0-10$

$= -10$

One-sided limits

Right hand limits

$$\lim_{x \to c^+} f(x)$$

limit as x approaches c from the right

Left hand limits

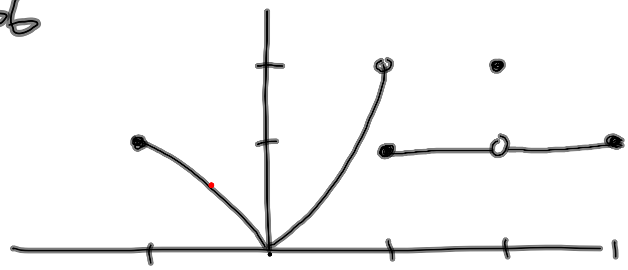
$$\lim_{x \to c^-} f(x)$$

limit as x approaches c from the left

*A function f(x) has a limit if and only if the left and right hand limits are equal.

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = \lim_{x \to c} f(x) = L$$

Ex 4) #38 p.66



True or False?

a) $\lim_{x\to -1^+} f(x) = 1$ True

b) $\lim_{x\to 2} f(x) =$ DNE False

c) $\lim_{x\to 2} f(x) = 2$ False

d) $\lim_{x\to 1^-} f(x) = 2$ True

e) $\lim_{x\to 1^+} f(x) = 1$ True

f) $\lim_{x\to 1} f(x) =$ DNE True

g) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$ True

h) $\lim_{x\to c} f(x)$ exists at every c in (-1,1) True

i) $\lim_{x\to c} f(x)$ exists at every c in (1,3). True

Sandwich / Squeeze Theorem

If $g(x) \leqslant f(x) \leqslant h(x)$

for values of c in an open interval and

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$$

THEN

$$\lim_{x\to c} f(x) = L$$

Ex 5) $\lim_{x \to 0} x \sin \frac{1}{x}$

$$-1 \le \sin x \le 1$$

$$x\left[-1 \le \sin \frac{1}{x} \le 1\right]$$

$$-1x \le x \sin \frac{1}{x} \le 1x$$

$$\lim_{x \to 0} -1x \le \lim_{x \to 0} x \sin \frac{1}{x} \le \lim_{x \to 0} x$$

$$0 \le 0 \le 0$$