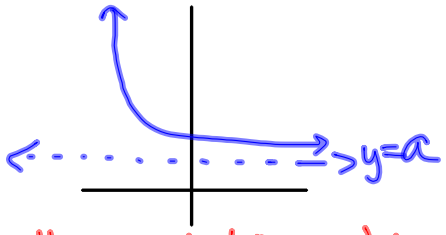
2.2 Limits Involving Infinity

Day 1

$\lim_{x \to \infty} f(x) = a$



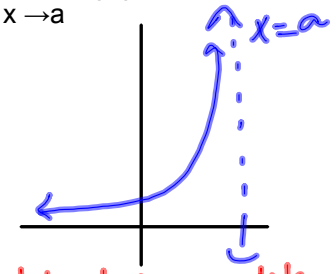
Horizontal Asymptote

The line $y=a$ is a H.A. of the graph of a function $y=f(x)$ if either

$\lim_{x \to \infty} f(x) = a$ or

$\lim_{x \to -\infty} f(x) = a$.

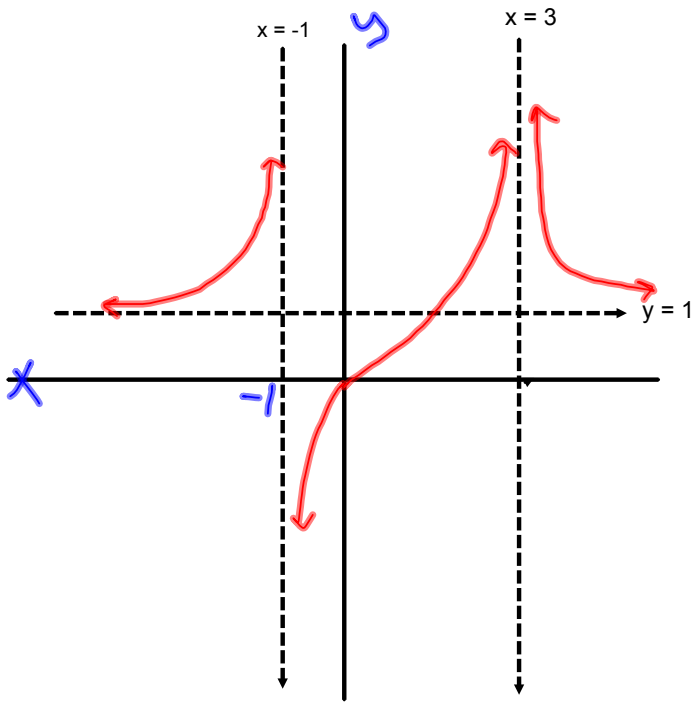
$\lim_{x \to a} f(x) = \infty$



Vertical Asymptote

The line $x=a$ is a V.A. of the graph of a function $y=f(x)$ if either $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$.

The graph of f(x) is below



Ex 1) $\lim_{x \to \infty} f(x) =$ 1

$\lim_{x \to -\infty} f(x) =$ 1

$\lim_{x \to -1^-} f(x) =$ ∞ or DNE

$\lim_{x \to -1^+} f(x) =$ $-\infty$ or DNE

$\lim_{x \to -1} f(x) =$ DNE

$\lim_{x \to 3^-} f(x) =$ ∞ or DNE

$\lim_{x \to 3^+} f(x) =$ ∞ or DNE

Vertical Asymptotes

How are they found? Zeros of denominator But not the factors that cancel out (holes)

Rational Function is in the form: $\dfrac{ax^n + bx^{n-1} + cx^{n-2}}{ax^n bx^{n-1} cx^{n-2}}$

Ex 2) Find the vertical asymptotes. $\dfrac{x+2}{x^2 - 5x + 4} = \dfrac{x+2}{(x-4)(x-1)}$

$x=4, x=1$

Horizontal Asymptotes / End Behavior

n = degree of numerator

m = degree of denominator

$f(x) = \dfrac{x + 2}{x^2 + 2x + 7}$ HA $n < m$ $y=0$ $\lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$

$f(x) = \dfrac{2x^2 + 7}{3x^2 - 5x + 2}$ HA $n = m$ $y = \frac{2}{3}$ $\lim_{x\to\infty} \frac{2x^2}{3x^2} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$

$f(x) = \dfrac{x^3 + 2x + 1}{x - 2}$ HA $n > m$ None $\lim_{x\to\infty} \frac{x^3}{x} = \lim_{x\to\infty} x^2 = \infty$

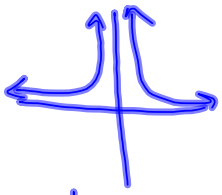
IMPORTANT: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Graph:

Memorize

Note: $\lim_{x \to \infty} f(x) = 0$

Also:



$\lim_{x \to 0} f(x) = \infty$ or DNE or No limit