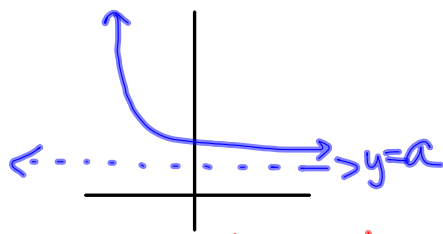


2.2 Limits Involving Infinity

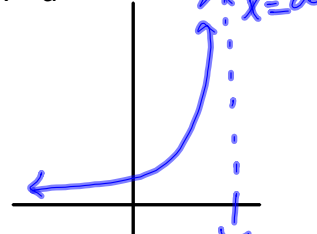
Day 1

$$\lim_{x \rightarrow \infty} f(x) = a$$



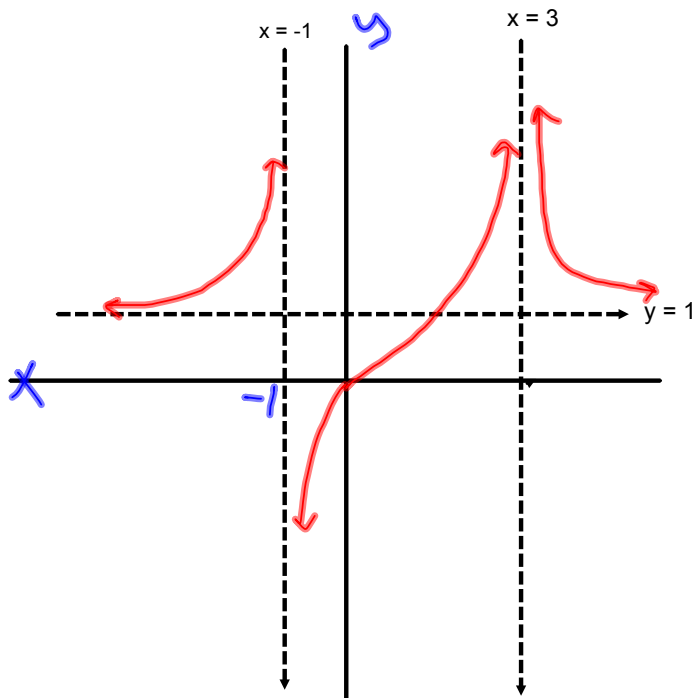
Horizontal Asymptote
 The line $y=a$ is a H.A. of the graph of a function $y=f(x)$ if either
 $\lim_{x \rightarrow \infty} f(x) = a$ or
 $\lim_{x \rightarrow -\infty} f(x) = a$.

$$\lim_{x \rightarrow a} f(x) = \infty$$



Vertical Asymptote
 The line $x=a$ is a V.A. of the graph of a function $y=f(x)$ if either
 $\lim_{x \rightarrow a^+} f(x) = \pm\infty$
 or
 $\lim_{x \rightarrow a^-} f(x) = \pm\infty$.

The graph of $f(x)$ is below



Ex 1) $\lim_{x \rightarrow \infty} f(x) = 1$

$\lim_{x \rightarrow -\infty} f(x) = 1$

$\lim_{x \rightarrow -1^-} f(x) = \infty$ or DNE

$\lim_{x \rightarrow -1^+} f(x) = -\infty$ or DNE

$\lim_{x \rightarrow -1} f(x) = \text{DNE}$

$\lim_{x \rightarrow 3^-} f(x) = \infty$ or DNE

$\lim_{x \rightarrow 3^+} f(x) = \infty$ or DNE

Vertical Asymptotes How are they found? *zeros of denominator But not the factors that cancel out (holes)*

Rational Function is in the form:
$$\frac{ax^n + bx^{n-1} + cx^{n-2}}{ax^n bx^{n-1} cx^{n-2}}$$

Ex 2) Find the vertical asymptotes. $\frac{x+2}{x^2 - 5x + 4} = \frac{x+2}{(x-4)(x-1)}$

$x=4, x=1$

Horizontal Asymptotes / End Behavior $n = \text{degree of numerator}$ $m = \text{degree of denominator}$

$f(x) = \frac{x+2}{x^2 + 2x + 7}$

HA $n < m$
 $y=0$

$\lim_{x \rightarrow \infty} \frac{x}{x^2} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$

$f(x) = \frac{2x^2 + 7}{3x^2 - 5x + 2}$

HA $n = m$
 $y = \frac{2}{3}$

$\lim_{x \rightarrow \infty} \frac{2x^2}{3x^2} = \lim_{x \rightarrow \infty} \frac{2}{3} = \frac{2}{3}$

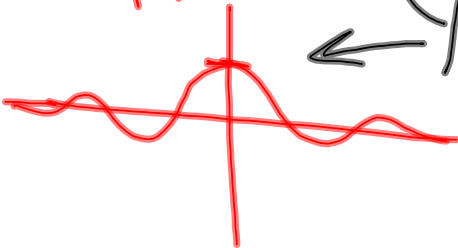
$f(x) = \frac{x^3 + 2x + 1}{x - 2}$

HA $n > m$
None

$\lim_{x \rightarrow \infty} \frac{x^3}{x} = \lim_{x \rightarrow \infty} x^2 = \infty$

IMPORTANT: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

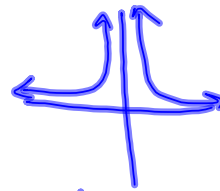
Graph:



Memorize

Note: $\lim_{x \rightarrow \infty} f(x) = 0$

Also:



$\lim_{x \rightarrow 0} f(x) = \infty$
or
DNE
or
No limit