Section 3.1 Exercises

1. $\frac{d}{dx}f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{-h}{x^2+hx}\,\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-h}{x^2h+h^2x} = \lim_{h\to 0} -\frac{1}{x^2+hx}$$

$$= -\frac{1}{x^2} - \frac{1}{(2)^2} = -\frac{1}{4}$$

3. $\frac{d}{dx}f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{3-(x+h)^2-(3-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{-2xh-h^2}{h}$$

$$= \lim_{h\to 0} -2x-h = -2x$$

$$-2(-1) = 2$$

5. $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$$= \lim_{x\to a} \frac{\frac{1}{x} - \frac{1}{a}}{x-a}$$

$$= \lim_{x\to a} \frac{a-x}{ax}\left(\frac{1}{x-a}\right)$$

$$= \lim_{x\to a} -\frac{1}{ax} = -\frac{1}{a^2}$$

$$-\frac{1}{(2)^2} = -\frac{1}{4}$$

7. $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$$= \lim_{x\to a} \frac{\sqrt{x+1}-\sqrt{a+1}}{x-a}$$

$$= \lim_{x\to a} \frac{x-a}{x-a(\sqrt{x+1}+\sqrt{a+1})}$$

$$= \lim_{x\to a} \frac{1}{2\sqrt{a+1}} = \frac{1}{2\sqrt{3+1}}$$

$$= \frac{1}{4}$$

9. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{[3(x+h)-12]-(3x-12)}{h}$$

$$= \lim_{h\to 0} \frac{3h}{h} = \lim_{h\to 0} 3 = 3$$

10. $\frac{dy}{dx} = \lim_{h\to 0} \frac{y(x+h)-y(x)}{h}$

$$= \lim_{h\to 0} \frac{7(x+h)-7x}{h}$$

$$= \lim_{h\to 0} \frac{7h}{h} = \lim_{h\to 0} 7 = 7$$

11. Let $f(x) = x^2$

$$\frac{d}{dx}(x^2) = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$$

$$= \lim_{h\to 0} (2x+h) = 2x$$

13. The graph of $y = f_1(x)$ is decreasing for $x < 0$ and increasing for $x > 0$, so its derivative is negative for $x < 0$ and positive for $x > 0$. (b)

14. The graph of $y = f_2(x)$ is always increasing, so its derivative is always ≥ 0. (a)

15. The graph of $y = f_3(x)$ oscillates between increasing and decreasing, so its derivative oscillates between positive and negative. (d)

16. The graph of $y = f_4(x)$ is decreasing, then increasing, then decreasing, and then increasing, so its derivative is negative, then positive, then negative, and then positive. (c)

17. **(a)** The tangent line has slope 5 and passes through (2, 3).

$$y = 5(x-2)+3$$

$$y = 5x-7$$

19. Let $f(x) = x^3$.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1+h)^3-1^3}{h}$$

$$= \lim_{h\to 0} \frac{1+3h+3h^2+h^3-1}{h}$$

$$= \lim_{h\to 0} (3+3h+h^2) = 3$$

(a) The tangent line has slope 3 and passes through (1, 1). Its equation is $y = 3(x-1)+1$, or $y = 3x-2$.

(b) The normal line has slope $-\frac{1}{3}$ and passes through (1, 1).

Its equation is $y = -\frac{1}{3}(x-1)+1$, or $y = -\frac{1}{3}x+\frac{4}{3}$.

21. (a) The amount of daylight is increasing at the fastest rate when the slope of the graph is largest. This occurs about one-fourth of the way through the year, sometime around April 1. The rate at this time is approximately $\frac{4 \text{ hours}}{24 \text{ days}}$ or $\frac{1}{6}$ hour per day.

(b) Yes, the rate of change is zero when the tangent to the graph is horizontal. This occurs near the beginning of the year and halfway through the year, around January 1 and July 1.

(c) Positive: January 1 through July 1
Negative: July 1 through December 31

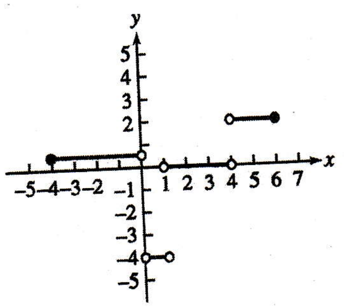
26. (a) The slope from $x = -4$ to $x = 0$ is $\frac{2-0}{0-(-4)} = \frac{1}{2}$.

The slope from $x = 0$ to $x = 1$ is $\frac{-2-2}{1-0} = -4$.

The slope from $x = 1$ to $x = 4$ is $\frac{-2-(-2)}{4-1} = 0$.

The slope from $x = 4$ to $x = 6$ is $\frac{2-(-2)}{6-4} = 2$.

Note that the derivative is undefined at $x = 0$, $x = 1$, and $x = 4$. (The function is differentiable at $x = -4$ and at $x = 6$ because these are endpoints of the domain and the one-sided derivatives exist.) The graph of the derivative is shown.



(b) $x = 0, 1, 4$

27. For $x > -1$, the graph of $y = f(x)$ must lie on a line of slope -2 that passes through $(0, -1)$: $y = -2x - 1$. Then $y(-1) = -2(-1) - 1 = 1$, so for $x < -1$, the graph of $y = f(x)$ must lie on a line of slope 1 that passes through $(-1, 1)$: $y = 1(x + 1) + 1$ or $y = x + 2$.

Thus $f(x) = \begin{cases} x+2, & x < -1 \\ -2x-1, & x \geq -1 \end{cases}$