Section 3.2 Exercises

1. Left-hand derivative:

$$\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{h^2-0}{h} = \lim_{h\to 0^-} h = 0$$

Right-hand derivative:

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{h-0}{h} = \lim_{h\to 0^+} 1 = 1$$

Since $0 \neq 1$, the function is not differentiable at the point P.

3. Left-hand derivative:

$$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{\sqrt{1+h}-1}{h}$$
$$= \lim_{h\to 0^-} \frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)}$$
$$= \lim_{h\to 0^-} \frac{(1+h)-1}{h(\sqrt{1+h}+1)}$$
$$= \lim_{h\to 0^-} \frac{1}{\sqrt{1+h}+1} = \frac{1}{2}$$

Right-hand derivative:

$$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{[2(1+h)-1]-1}{h}$$
$$= \lim_{h\to 0^+} \frac{2h}{h}$$
$$= \lim_{h\to 0^+} 2 = 2$$

5. (a) All points in $[-3, 2]$

(b) None

(c) None

7. (a) All points in $[-3, 3]$ except $x = 0$

(b) None

(c) $x = 0$

8. (a) All points in $[-2, 3]$ except $x = -1, 0, 2$

(b) $x = -1$

(c) $x = 0, x = 2$

9. (a) All points in $[-1, 2]$ except $x = 0$

13. Note that $y = x + \sqrt{x^2} + 2 = x + |x| + 2$

$$= \begin{cases} 2, & x \le 0 \\ 2x+2, & x > 0. \end{cases}$$

$$\lim_{h\to 0^-} \frac{y(0+h)-y(0)}{h} = \lim_{h\to 0^-} \frac{2-2}{h} = \lim_{h\to 0^-} 0 = 0$$
$$\lim_{h\to 0^+} \frac{y(0+h)-y(0)}{h} = \lim_{h\to 0^+} \frac{(2h+2)-2}{h} = \lim_{h\to 0^+} 2 = 2$$

The problem is a corner.

15. Note that $y = 3x - 2|x| - 1 = \begin{cases} 5x-1, & x \le 0 \\ x-1, & x > 0 \end{cases}$

$$\lim_{h\to 0^-} \frac{y(0+h)-y(0)}{h} = \lim_{h\to 0^-} \frac{(5h-1)-(-1)}{h} = \lim_{h\to 0^-} 5 = 5$$
$$\lim_{h\to 0^+} \frac{y(0+h)-y(0)}{h} = \lim_{h\to 0^+} \frac{(h-1)-(-1)}{h} = \lim_{h\to 0^+} 1 = 1$$

The problem is a corner.

17. $\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$

$= \frac{4(0.001)-(0.001)^2-(4(-0.001)-(-0.001)^2)}{0.002}$

$= 4$, yes it is differentiable.

19. $\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$

$= \frac{4(1.001)+(1.001)^2-4(0.999)-(0.99)^2}{0.002}$

$= 2$, yes it is differentiable.

21. $\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$

$= \frac{(-1.999)^3-4(-1.999)-((-2.001)^3-4(-2.001))}{0.002}$

$= 8.000001$, yes it is differentiable.

31. Find the zeros of the denominator.

$x^2-4x-5=0$

$(x+1)(x-5)=0$

$x=-1$ or $x=5$

The function is a rational function, so it is differentiabe for all x in its domain: all reals except $x=-1, 5$.

33. Note that the sine function is odd, so

$$P(x)=\sin(|x|)-1=\begin{cases}-\sin x-1, & x<0\\ \sin x-1, & x\ge 0.\end{cases}$$

The graph of $P(x)$ has a corner at $x=0$. The function is differentiable for all reals except $x=0$.

35. The function is piecewise-defined in terms of polynomials, so it is differentiable everywhere except possibly at $x=0$ and at $x=3$. Check $x=0$:

$\lim_{h\to 0^-}\frac{g(0+h)-g(0)}{h}=\lim_{h\to 0^-}\frac{(h+1)^2-1}{h}=\lim_{h\to 0^-}\frac{h^2+2h}{h}$

$=\lim_{h\to 0^-}(h+2)=2$

$\lim_{h\to 0^+}\frac{g(0+h)-g(0)}{h}=\lim_{h\to 0^+}\frac{(2h+1)-1}{h}=\lim_{h\to 0^+}2=2$

The function is differentiable at $x=0$.

Check $x=3$:

Since $g(3)=(4-3)^2=1$ and

$\lim_{x\to 3^-}g(x)=\lim_{x\to 3^-}(2x+1)=2(3)+1=7$, the function is not continuous (and hence not differentiable) at $x=3$. The function is differentiable for all reals except $x=3$.

Section 3.3 Exercises

1. $\frac{dy}{dx}=\frac{d}{dx}(-x^2)+\frac{d}{dx}(3)=-2x+0=-2x$

3. $\frac{dy}{dx}=\frac{d}{dx}(2x)+\frac{d}{dx}(1)=2+0=2$

5. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{3}x^3\right)+\frac{d}{dx}\left(\frac{1}{2}x^2\right)+\frac{d}{dx}(x)$

$=x^2+x+1$

7. $\frac{dy}{dx}=\frac{d}{dx}(x^3-2x^2+x+1)$

$=3x^2-4x+1=0$

$x=\frac{1}{3}, 1$