Section 3.3 Exercises

1. $\frac{dy}{dx} = \frac{d}{dx}(-x^2) + \frac{d}{dx}(3) = -2x + 0 = -2x$

3. $\frac{dy}{dx} = \frac{d}{dx}(2x) + \frac{d}{dx}(1) = 2 + 0 = 2$

5. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3\right) + \frac{d}{dx}\left(\frac{1}{2}x^2\right) + \frac{d}{dx}(x)$

$= x^2 + x + 1$

7. $\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x^2 + x + 1)$

$= 3x^2 - 4x + 1 = 0$

$x = \frac{1}{3},\ 1$

9. $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 4x^2 + 1)$

$= 4x^3 - 8x = 0$

$x = 0,\ \pm\sqrt{2}$

13. (a) $\frac{dy}{dx} = \frac{d}{dx}[(x+1)(x^2+1)]$

$= (x+1)\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}(x+1)$

$= (x+1)(2x) + (x^2+1)(1)$

$= 2x^2 + 2x + x^2 + 1$

$= 3x^2 + 2x + 1$

13. Continued

(b) $\frac{dy}{dx} = \frac{d}{dx}[(x+1)(x^2+1)]$

$= \frac{d}{dx}(x^3 + x^2 + x + 1)$

$= 3x^2 + 2x + 1$

15. $(x^3 + x + 1)(x^4 + x^2 + 1)$

$\frac{d}{dx}(x^7 + 2x^5 + x^4 + 2x^3 + x^2 + x + 1)$

$= 7x^6 + 10x^4 + 4x^3 + 6x^2 + 2x + 1$

17. $\frac{dy}{dx} = \frac{d}{dx}\frac{2x+5}{3x-2} = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2} = -\frac{19}{(3x-2)^2}$

19. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(x-1)(x^2+x+1)}{x^3}\right) = \frac{d}{dx}\left(\frac{x^3-1}{x^3}\right)$

$= \frac{d}{dx}(1 - x^{-3}) = 0 + 3x^{-4} = \frac{3}{x^4}$

21. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2}{1-x^3}\right) = \frac{(1-x^3)(2x) - x^2(-3x^2)}{(1-x^3)^2} = \frac{x^4 + 2x}{(1-x^3)^2}$

23. (a) At $x = 0$, $\frac{d}{dx}(uv) = u(0)v'(0) + v(0)u'(0)$

$= (5)(2) + (-1)(-3) = 13$

(b) At $x = 0$, $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(0)u'(0) - u(0)v'(0)}{[v(0)]^2}$

$= \frac{(-1)(-3) - (5)(2)}{(-1)^2} = -7$

(c) At $x = 0$, $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u(0)v'(0) - v(0)u'(0)}{[u(0)]^2}$

$= \frac{(5)(2) - (-1)(-3)}{(5)^2} = \frac{7}{25}$

(d) At $x = 0$, $\frac{d}{dx}(7v - 2u) = 7v'(0) - 2u'(0)$

$= 7(2) - 2(-3) = 20$

25. $y'(x) = 2x + 5$

$y'(3) = 2(3) + 5 = 11$

The slope is 11. (iii)

27. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^3+1}{2x}\right)$

$= \frac{(3x^2)2x - 2(x^3+1)}{4x^2}$

$= \frac{4x^3 - 2}{4x^2}$

$y'(1) = \frac{4(1)^3 - 2}{4(1)^2} = \frac{1}{2}$

$y(1) = \frac{(1)^3 + 1}{2(1)} = 1$

$y = \frac{1}{2}(x-1) + 1 = \frac{1}{2}x + \frac{1}{2}$

29. $\frac{dy}{dx} = \frac{d}{dx}(4x^{-2} - 8x + 1)$

$= -8x^{-3} - 8$

35. $y = x^{-1} + x^2$

$y^{\mathrm{I}} = -x^{-2} + 2x$

$y^{\mathrm{II}} = 2x^{-3} + 2$

$y^{\mathrm{III}} = -6x^{-4}$

$y^{\mathrm{IV}} = -24x^{-5}$

37. $y'(x) = 3x^2 - 3$

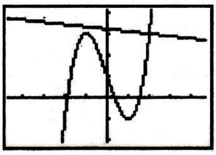
$y'(2) = 3(2)^2 - 3 = 9$

The tangent line has slope 9, so the perpendicular line has slope $-\frac{1}{9}$ and passes through (2, 3).

$y = -\frac{1}{9}(x-2) + 3$

$y = -\frac{1}{9}x + \frac{29}{9}$

Graphical support:



[−4.7, 4.7] by [−2.1, 4.1]

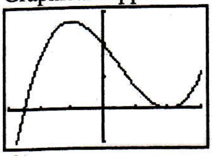
39. $y'(x) = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x+1)(x-2)$

The tangent is parallel to the x-axis when $y' = 0$, at $x = -1$ and at $x = 2$. Since $y(-1) = 27$ and $y(2) = 0$, the two points where this occurs are (−1, 27) and (2, 0).

Graphical support:



[−3, 3] by [−10, 30]