Section 3.4 Exercises

1. (a) $V(s) = s^3$

(b) $\frac{dv}{ds} = 3s^2$

(c) $V'(1) = 3(1)^2 = 3$

$V'(5) = 3(5)^2 = 75$

3. (a) $A(s) = \frac{\sqrt{3}}{4}s^2$

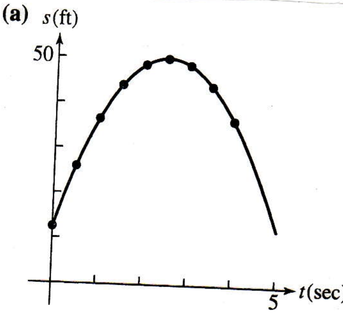
(b) $\frac{dA}{ds} = \frac{\sqrt{3}}{2}s$

(c) $A'(2) = \frac{\sqrt{3}}{2}(2) = \sqrt{3}$

$A'(10) = \frac{\sqrt{3}}{2}(10) = 5\sqrt{3}$

(d) $\frac{\text{in}^2}{\text{in}}$.

5. (a)



(b) $s'(1) = 18$, $s'(2.5) = 0$, $s'(3.5) = -12$

9. (a) The particle moves forward when $v > 0$, for $0 \le t < 1$ and for $5 < t < 7$.
The particle moves backward when $v < 0$, for $1 < t < 5$.
The particle speeds up when v is negative and decreasing, for $1 < t < 2$, and when v is positive and increasing, for $5 < t < 6$.
The particle slows down when v is positive and decreasing, for $0 \le t < 1$ and for $6 < t < 7$, and when v is negative and increasing, for $3 < t < 5$.

(b) Note that the acceleration $a = \frac{dv}{dt}$ is undefined at $t = 2$, $t = 3$, and $t = 6$.
The acceleration is positive when v is increasing, for $3 < t < 6$.
The acceleration is negative when v is decreasing, for $0 \le t < 2$ and for $6 < t < 7$.
The acceleration is zero when v is constant, for $2 < t < 3$ and for $7 < t \le 9$.

(c) The particle moves at its greatest speed when $|v|$ is maximized. at $t = 0$ and for $2 < t < 3$.

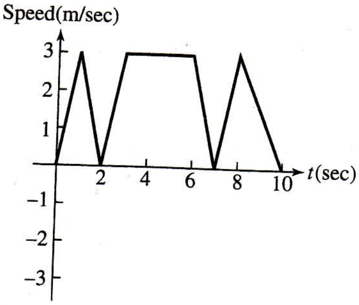
9. Continued

(d) The particle stands still for more than an instant when v stays at zero, for $7 < t \le 9$.

11. **(a)** The body reverses direction when v changes sign, at $t = 2$ and at $t = 7$.

(b) The body is moving at a constant speed, $|v| = 3$ m/sec, between $t = 3$ and $t = 6$.

(c) The speed graph is obtained by reflecting the negative portion of the velocity graph, $2 < t < 7$, about the x-axis.



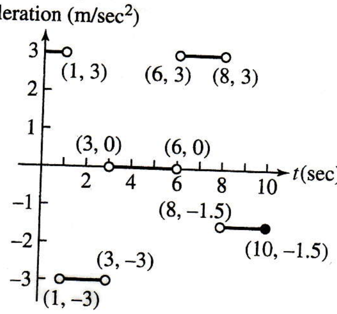
(d) For $0 \le t < 1$: $a = \dfrac{3-0}{1-0} = 3$ m/sec^2

For $1 < t < 3$: $a = \dfrac{-3-3}{3-1} = -3$ m/sec^2

For $3 < t < 6$: $a = \dfrac{-3-(-3)}{6-3} = 0$ m/sec^2

For $6 < t < 8$: $a = \dfrac{3-(-3)}{8-6} = 3$ m/sec^2

For $8 < t \le 10$: $a = \dfrac{0-3}{10-8} = -1.5$ m/sec^2



13. **(a)** Velocity: $v(t) = \dfrac{ds}{dt} = \dfrac{d}{dt}(24t - 0.8t^2) = 24 - 1.6t$ m/sec

Acceleration: $a(t) = \dfrac{dv}{dt} = \dfrac{d}{dt}(24 - 1.6t) = -1.6$ m/sec^2

(b) The rock reaches its highest point when $v(t) = 24 - 1.6t = 0$, at $t = 15$. It took 15 seconds.

(c) The maximum height was $s(15) = 180$ meters.

(d)

$$s(t) = \frac{1}{2}(180)$$
$$24t - 0.8t^2 = 90$$
$$0 = 0.8t^2 - 24t + 90$$
$$t = \frac{24 \pm \sqrt{(-24)^2 - 4(0.8)(90)}}{2(0.8)}$$
$$\approx 4.393,\ 25.607$$

It took about 4.393 seconds to reach half its maximum height.

(e)

$$s(t) = 0$$
$$24t - 0.8t^2 = 0$$
$$0.8t(30 - t) = 0$$
$$t = 0 \text{ or } t = 30$$

The rock was aloft from $t = 0$ to $t = 30$, so it was aloft for 30 seconds.

15. The rock reaches its maximum height when the velocity $s'(t) = 24 - 9.8t = 0$, at $t \approx 2.449$. Its maximum height is about $s(2.449) \approx 29.388$ meters.

19. **(a)** Displacement: $= s(5) - s(0) = 12 - 2 = 10$ m

(b) Average velocity $= \dfrac{10 \text{ m}}{5 \text{ sec}} = 2$ m/sec

(c) Velocity $= s'(t) = 2t - 3$
At $t = 4$, velocity $= s'(4) = 2(4) - 3 = 5$ m/sec

(d) Acceleration $= s''(t) = 2$ m/sec^2

(e) The particle changes direction when $s'(t) = 2t - 3 = 0$, so $t = \dfrac{3}{2}$ sec.

(f) Since the acceleration is always positive, the position s is at a minimum when the particle changes direction, at $t = \dfrac{3}{2}$ sec. Its position at this time is $s\left(\dfrac{3}{2}\right) = -\dfrac{1}{4}$ m.

21. **(a)** $v(t) = \dfrac{ds}{dt} = \dfrac{d}{dt}(t-2)^2(t-4)$
$= (t-2)(3t-10)$

(b) $a(t) = \dfrac{dv}{dt} = \dfrac{d}{dt}(t-2)(3t-10)$
$a(t) = 6t - 16$

(c) $v(t) = (t-2)(3t-10) = 0$
$t = 2,\ \dfrac{10}{3}$

(d) The particle starts at the point $s = -16$ when $t = 0$ and move right until it stops at $s = 0$ when $t = 2$, then it moves left to the point $s = -1.185$ when $t = \dfrac{10}{3}$ where it stops again, and finally continues right from there on.

23. $v(t) = s'(t) = 3t^2 - 12t + 9$
$a(t) = v'(t) = 6t - 12$
Find when velocity is zero.
$3t^2 - 12t + 9 = 0$
$3(t^2 - 4t + 3) = 0$
$3(t-1)(t-3) = 0$
$t = 1$ or $t = 3$
At $t = 1$, the acceleration is $a(1) = -6$ m/sec^2
At $t = 3$, the acceleration is $a(3) = 6$ m/sec^2

Section 3.5 Exercises

1. $\dfrac{d}{dx}(1 + x - \cos x) = 0 + 1 - (-\sin x) = 1 + \sin x$

3. $\dfrac{d}{dx}\left(\dfrac{1}{x} + 5\sin x\right) = -\dfrac{1}{x^2} + 5\cos x$

5. $\dfrac{d}{dx}(4 - x^2 \sin x) = \dfrac{d}{dx}(4) - \left[x^2 \dfrac{d}{dx}(\sin x) + (\sin x)\dfrac{d}{dx}(x^2)\right]$
$= 0 - [x^2 \cos x + (\sin x)(2x)]$
$= -x^2 \cos x - 2x \sin x$

7. $\dfrac{d}{dx}\left(\dfrac{4}{\cos x}\right) = \dfrac{d}{dx}(4 \sec x) = 4 \sec x \tan x$