Section 3.5 Exercises

1. $\dfrac{d}{dx}(1+x-\cos x)=0+1-(-\sin x)=1+\sin x$

3. $\dfrac{d}{dx}\left(\dfrac{1}{x}+5\sin x\right)=-\dfrac{1}{x^2}+5\cos x$

5. $\dfrac{d}{dx}(4-x^2\sin x)=\dfrac{d}{dx}(4)-\left[x^2\dfrac{d}{dx}(\sin x)+(\sin x)\dfrac{d}{dx}(x^2)\right]$

$=0-[x^2\cos x+(\sin x)(2x)]$

$=-x^2\cos x-2x\sin x$

7. $\dfrac{d}{dx}\left(\dfrac{4}{\cos x}\right)=\dfrac{d}{dx}(4\sec x)=4\sec x\tan x$

15. (a) $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dt}(2\sin t+3\cos t)$

$v(t)=2\cos t-3\sin t$, speed $=|2\cos t-3\sin t|$

$a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}(2\cos t-3\sin t)$

$a(t)=-2\sin t-3\cos t$

(b) $v\left(\dfrac{\pi}{4}\right)=2\cos\dfrac{\pi}{4}-3\sin\dfrac{\pi}{4}$

$v\left(\dfrac{\pi}{4}\right)=-\dfrac{\sqrt{2}}{2}$

speed $=\dfrac{\sqrt{2}}{2}$

$a\left(\dfrac{\pi}{4}\right)=-2\sin\dfrac{\pi}{4}-3\cos\dfrac{\pi}{4}$

$a\left(\dfrac{\pi}{4}\right)=\dfrac{-5\sqrt{2}}{2}$

(c) The body starts at 3, goes to $3.606(\sqrt{13})$, and then oscillates between –3.606 and 3.606. The period of the motion is 2π.

13. (a) $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dt}(2+3\sin t)$

$v(t)=3\cos t$, speed $=|3\cos t|$

$a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}(3\cos t)=-3\sin t$

(b) $v\left(\dfrac{\pi}{4}\right)=3\cos\left(\dfrac{\pi}{4}\right)=\dfrac{3\sqrt{2}}{2}$, speed $=\dfrac{3\sqrt{2}}{2}$

$a\left(\dfrac{\pi}{4}\right)=-3\sin\left(\dfrac{\pi}{4}\right)=-\dfrac{3\sqrt{2}}{2}$

(c) The body starts at 2, goes up to 5, goes down to –1, and then oscillates between –1 and 5. The period of motion is 2π.

22. $y=\sec x$

$\dfrac{dy}{dx}=\dfrac{d}{dx}(\sec x)=\sec x\tan x$

$y\left(\dfrac{\pi}{4}\right)=\sec\dfrac{\pi}{4}=1.414$

$y'\left(\dfrac{\pi}{4}\right)=\sec\dfrac{\pi}{4}\tan\dfrac{\pi}{4}=1.414$

tangent: $y=1.414\left(x-\dfrac{\pi}{4}\right)+1.414$

$y=1.414x+0.303$

normal: $m_2=-\dfrac{1}{m_1}=-0.707$

$y=-0.707\left(x-\dfrac{\pi}{4}\right)+1.414=-0.707x+1.970$

23. $y=x^2\sin x$

$\dfrac{dy}{dx}=\dfrac{d}{dx}(x^2\sin x)=2x\sin x+x^2\cos x$

$y(3)=(3)^2\sin 3=1.270$

$y'(3)=2(3)\sin 3+(3)^2\cos 3=-8.063$

tangent: $y=-8.063(x-3)+1.270=-8.063x+25.460$

normal: $m_2=-\dfrac{1}{m_1}=0.124$

$y=0.124(x-3)+1.270$

$y=0.124x+0.898$

31. $y'(x)=\dfrac{d}{dx}(4+\cot x-2\csc x)$

$=0-\csc^2 x+2\csc x\cot x$

$=-\csc^2 x+2\csc x\cot x$

(a) $y'\left(\dfrac{\pi}{2}\right)=-\csc^2\dfrac{\pi}{2}+2\csc\dfrac{\pi}{2}\cot\dfrac{\pi}{2}$

$=-1^2+2(1)(0)=-1$

The tangent line has slope –1 and passes through $P\left(\dfrac{\pi}{2},\ 2\right)$. Its equation is $y=-1\left(x-\dfrac{\pi}{2}\right)+2$, or

$y=-x+\dfrac{\pi}{2}+2$.

(b) $f'(x)=0$

$-\csc^2 x+2\csc x\cot x=0$

$-\dfrac{1}{\sin^2 x}+\dfrac{2\cos x}{\sin^2 x}=0$

$\dfrac{1}{\sin^2 x}(2\cos x-1)=0$

$\cos x=\dfrac{1}{2}$

$x=\dfrac{\pi}{3}$ at point Q

$y\left(\dfrac{\pi}{3}\right)=4+\cot\dfrac{\pi}{3}-2\csc\dfrac{\pi}{3}$

$=4+\dfrac{1}{\sqrt{3}}-2\left(\dfrac{2}{\sqrt{3}}\right)$

$=4-\dfrac{3}{\sqrt{3}}=4-\sqrt{3}$

The coordinates of Q are $\left(\dfrac{\pi}{3},\ 4-\sqrt{3}\right)$.

The equation of the horizontal line is $y=4-\sqrt{3}$.

32. $y'(x)=\dfrac{d}{dx}(1+\sqrt{2}\csc x+\cot x)$

$=0+\sqrt{2}(-\csc x\cot x)+(-\csc^2 x)$

$=-\sqrt{2}\csc x\cot x-\csc^2 x$

(a) $y'\left(\dfrac{\pi}{4}\right)=-\sqrt{2}\csc\dfrac{\pi}{4}\cot\dfrac{\pi}{4}-\csc^2\dfrac{\pi}{4}$

$=-\sqrt{2}(\sqrt{2})(1)-(\sqrt{2})^2$

$=-2-2=-4$

The tangent line has slope –4 and passes through $P\left(\dfrac{\pi}{4},\ 4\right)$. Its equation is $y=-4\left(x-\dfrac{\pi}{4}\right)+4$, or

$y=-4x+\pi+4$.

(b) $y'(x)=0$

$-\sqrt{2}\csc x\cot x-\csc^2 x=0$

$-\dfrac{\sqrt{2}\cos x}{\sin^2 x}-\dfrac{1}{\sin^2 x}=0$

$-\dfrac{1}{\sin^2 x}(\sqrt{2}\cos x+1)=0$

$\cos x=-\dfrac{1}{\sqrt{2}}$

$x=\dfrac{3\pi}{4}$ at point Q

$y\left(\dfrac{3\pi}{4}\right)=1+\sqrt{2}\csc\dfrac{3\pi}{4}+\cot\dfrac{3\pi}{4}$

$=1+\sqrt{2}(\sqrt{2})+(-1)$

$=2$

The coordinates of Q are $\left(\dfrac{3\pi}{4},\ 2\right)$.

The equation of the horizontal line is $y=2$.