Section 3.6 Exercises

1. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\sin u \qquad u=3x+1$

$\frac{dy}{du}=\cos u \qquad \frac{du}{dx}=3$

$\frac{dy}{dx}=3\cos(3x+1)$

3. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\cos u \qquad u=\sqrt{3}x$

$\frac{dy}{du}=-\sin u \qquad \frac{du}{dx}=\sqrt{3}$

$\frac{dy}{dx}=-\sqrt{3}\sin(\sqrt{3}x)$

5. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=u^2 \qquad u=\frac{\sin x}{1+\cos x}$

$\frac{dy}{du}=2u \qquad \frac{du}{dx}=\frac{\sin x}{(1+\cos x)^2}$

$\frac{dy}{dx}=\frac{2\sin x}{(1+\cos x)^2}$

7. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\cos u \qquad u=\sin x$

$\frac{dy}{du}=-\sin u \qquad \frac{du}{dx}=\cos x$

$\frac{dy}{dx}=-\sin(\sin x)\cos x$

13. $\frac{dy}{dx}=\frac{d}{dx}(x+\sqrt{x})^{-2}=-2(x+\sqrt{x})^{-3}\frac{d}{dx}(x+\sqrt{x})$

$=-2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$

15. $\frac{dy}{dx}=\frac{d}{dx}(\sin^{-5}x-\cos^3 x)$

$=(-5\sin^{-6}x)\frac{d}{dx}(\sin x)-(3\cos^2 x)\frac{d}{dx}(\cos x)$

$=-5\sin^{-6}x\cos x+3\cos^2 x\sin x$

17. $\frac{dy}{dx}=\frac{d}{dx}(\sin^3 x\tan 4x)$

$=(\sin^3 x)\frac{d}{dx}(\tan 4x)+(\tan 4x)\frac{d}{dx}(\sin^3 x)$

$=(\sin^3 x)(\sec^2 4x)\frac{d}{dx}(4x)+(\tan 4x)(3\sin^2 x)\frac{d}{dx}(\sin x)$

$=(\sin^3 x)(\sec^2 4x)(4)+(\tan 4x)(3\sin^2 x)(\cos x)$

$=4\sin^3 x\sec^2 4x+3\sin^2 x\cos x\tan 4x$

19. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3}{\sqrt{2x+1}}\right)$

$=\frac{(\sqrt{2x+1})\frac{d}{dx}(3)-3\frac{d}{dx}(\sqrt{2x+1})}{(\sqrt{2x+1})^2}$

$=\frac{(\sqrt{2x+1})(0)-3\left(\frac{1}{2\sqrt{2x+1}}\right)\frac{d}{dx}(2x+1)}{2x+1}$

$=\frac{-3\left(\frac{1}{2\sqrt{2x+1}}\right)(2)}{2x+1}$

$=-\frac{3}{(2x+1)\sqrt{2x+1}}$

$=-3(2x+1)^{-3/2}$

21. The last step here uses the identity 2 sin *a* cos *a* = sin 2*a*.

$\frac{dy}{dx}=\frac{d}{dx}\sin^2(3x-2)$

$=2\sin(3x-2)\frac{d}{dx}\sin(3x-2)$

$=2\sin(3x-2)\cos(3x-2)\frac{d}{dx}(3x-2)$

$=2\sin(3x-2)\cos(3x-2)(3)$

$=6\sin(3x-2)\cos(3x-2)$

$=3\sin(6x-4)$

23. $\frac{dy}{dx}=\frac{d}{dx}(1+\cos^2 7x)^3$

$=3(1+\cos^2 7x)^2\frac{d}{dx}(1+\cos^2 7x)$

$=3(1+\cos^2 7x)^2(2\cos 7x)\frac{d}{dx}(\cos 7x)$

$=3(1+\cos^2 7x)^2(2\cos 7x)(-\sin 7x)\frac{d}{dx}(7x)$

$=3(1+\cos^2 7x)^2(2\cos 7x)(-\sin 7x)(7)$

$=-42(1+\cos^2 7x)^2\cos 7x\sin 7x$

25. $\frac{dr}{d\theta}=\frac{d}{d\theta}\tan(2-\theta)=\sec^2(2-\theta)\frac{d}{d\theta}(2-\theta)$

$=\sec^2(2-\theta)(-1)=-\sec^2(2-\theta)$

27. $\frac{dr}{d\theta}=\frac{d}{d\theta}\sqrt{\theta\sin\theta}=\frac{1}{2\sqrt{\theta\sin\theta}}\frac{d}{d\theta}(\theta\sin\theta)$

$=\frac{1}{2\sqrt{\theta\sin\theta}}\left[\theta\frac{d}{d\theta}(\sin\theta)+(\sin\theta)\frac{d}{d\theta}(\theta)\right]$

$=\frac{1}{2\sqrt{\theta\sin\theta}}(\theta\cos\theta+\sin\theta)$

$=\frac{\theta\cos\theta+\sin\theta}{2\sqrt{\theta\sin\theta}}$

29. $y'=\frac{d}{dx}\tan x=\sec^2 x$

$y''=\frac{d}{dx}\sec^2 x=(2\sec x)\frac{d}{dx}(\sec x)$

$=(2\sec x)(\sec x\tan x)$

$=2\sec^2 x\tan x$

33. $f'(u)=\frac{d}{du}(u^5+1)=5u^4$

$g'(x)=\frac{d}{dx}(\sqrt{x})=\frac{1}{2\sqrt{x}}$

$(f\circ g)'(1)=f'(g(1))g'(1)=f'(1)g'(1)=(5)\left(\frac{1}{2}\right)=\frac{5}{2}$

41. $\frac{dx}{dt} = \frac{d}{dt}(2\cos t) = -2\sin t$

$\frac{dy}{dt} = \frac{d}{dt}(2\sin t) = 2\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{-2\sin t} = -\cot t$$

43. $\frac{dx}{dt} = \frac{d}{dt}(\sec^2 t - 1) = (2\sec t)\frac{d}{dt}(\sec t)$

$= (2\sec t)(\sec t \tan t)$

$= 2\sec^2 t \tan t$

$\frac{dy}{dt} = \frac{d}{dt}\tan t = \sec^2 t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{2\sec^2 t \tan t} = \frac{1}{2}\cot t.$$

The line passes through

$\left(\sec^2\left(-\frac{\pi}{4}\right) - 1,\ \tan\left(-\frac{\pi}{4}\right)\right) = (1,\ -1)$ and has

slope $\frac{1}{2}\cot\left(-\frac{\pi}{4}\right) = -\frac{1}{2}$. Its equation

is $y = -\frac{1}{2}(x-1) - 1$, or $y = -\frac{1}{2}x - \frac{1}{2}$.

47. $\frac{dx}{dt} = \frac{d}{dt}(t - \sin t) = 1 - \cos t$

$\frac{dy}{dt} = \frac{d}{dt}(1 - \cos t) = \sin t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{1 - \cos t}$$

The line passes through

$\left(\frac{\pi}{3} - \sin\frac{\pi}{3}, 1 - \cos\frac{\pi}{3}\right) = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and has slope

$$\frac{\sin\left(\frac{\pi}{3}\right)}{1 - \cos\left(\frac{\pi}{3}\right)} = \sqrt{3}.$$ Its equation is

$y = \sqrt{3}\left(x - \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) + \frac{1}{2}$, or

$y = \sqrt{3}x + 2 - \frac{\pi}{\sqrt{3}}$.

51. $\frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt} = \frac{d}{d\theta}(\cos\theta)\frac{d\theta}{dt}$

$= (-\sin\theta)\left(\frac{d\theta}{dt}\right)$

When $\theta = \frac{3\pi}{2}$ and $\frac{d\theta}{dt} = 5$, $\frac{ds}{dt} = \left(-\sin\frac{3\pi}{2}\right)(5) = 5$.