Section 3.7 Exercises

1.

$$x^2y+xy^2=6$$

$$\frac{d}{dx}(x^2y)+\frac{d}{dx}(xy^2)=\frac{d}{dx}(6)$$

$$x^2\frac{dy}{dx}+y(2x)+x(2y)\frac{dy}{dx}+y^2(1)=0$$

$$x^2\frac{dy}{dx}+2xy\frac{dy}{dx}=-(2xy+y^2)$$

$$(2xy+x^2)\frac{dy}{dx}=-(2xy+y^2)$$

$$\frac{dy}{dx}=-\frac{2xy+y^2}{2xy+x^2}$$

3. $y^2=\frac{x-1}{x+1}$

$$\frac{d}{dx}y^2=\frac{d}{dx}\frac{x-1}{x+1}$$

$$2y\frac{dy}{dx}=\frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$$

$$2y\frac{dy}{dx}=\frac{2}{(x+1)^2}$$

$$\frac{dy}{dx}=\frac{1}{y(x+1)^2}$$

7.

$$x+\tan xy=0$$

$$\frac{d}{dx}(x)+\frac{d}{dx}(\tan xy)=\frac{d}{dx}(0)$$

$$1+\sec^2(xy)\frac{d}{dx}(xy)=0$$

$$1+(\sec^2 xy)[x\frac{dy}{dx}+(y)(1)]=0$$

$$(\sec^2 xy)(x)\frac{dy}{dx}=-1-(\sec^2 xy)(y)$$

$$\frac{dy}{dx}=\frac{-1-y\sec^2 xy}{x\sec^2 xy}$$

$$\frac{dy}{dx}=-\frac{1}{x}\cos^2 xy-\frac{y}{x}$$

9. $\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(13)$

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{y},\ -\frac{-2}{3}=\frac{2}{3}$$

11. $\frac{d}{dx}((x-1)^2+(y-1)^2)=\frac{d}{dx}(13)$

$$2(x-1)1+(2(y-1)1)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x-1}{y-1},\ -\frac{3-1}{4-1}=-\frac{2}{3}$$

13. $\frac{d}{dx}(x^2y-xy^2)=\frac{d}{dx}(4)$

$$2xy-y^2+(2xy-x^2)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{2xy-y^2}{2xy-x^2},$$

defined at every point except where $x=0$ or $y=\frac{x}{2}$

17. $x^2 + xy - y^2 = 1$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) - \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + x\frac{dy}{dx} + (y)(1) - 2y\frac{dy}{dx} = 0$$

$$(x - 2y)\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x - 2y} = \frac{2x + y}{2y - x}$$

Slope at (2, 3): $\frac{2(2)+3}{2(1)-2} = \frac{7}{4}$

(a) Tangent: $y = \frac{7}{4}(x-2) + 3$ or $y = \frac{7}{4}x - \frac{1}{2}$

(b) Normal: $y = -\frac{4}{7}(x-2) + 3$ or $y = -\frac{4}{7}x + \frac{29}{7}$

19. $x^2y^2 = 9$

$$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}(9)$$

$$(x^2)(2y)\frac{dy}{dx} + (y^2)(2x) = 0$$

$$2x^2y\frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx} = -\frac{2xy^2}{2x^2y} = -\frac{y}{x}$$

Slope at (−1, 3): $-\frac{3}{-1} = 3$

(a) Tangent: $y = 3(x+1) + 3$ or $y = 3x + 6$

(b) Normal: $y = -\frac{1}{3}(x+1) + 3$ or $y = -\frac{1}{3}x + \frac{8}{3}$

21. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$

$$\frac{d}{dx}(6x^2) + \frac{d}{dx}(3xy) + \frac{d}{dx}(2y^2) + \frac{d}{dx}(17y) - \frac{d}{dx}(6) = \frac{d}{dx}(0)$$

$$12x + 3x\frac{dy}{dx} + (3y)(1) + 4y\frac{dy}{dx} + 17\frac{dy}{dx} - 0 = 0$$

$$3x\frac{dy}{dx} + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = -12x - 3y$$

$$(3x + 4y + 17)\frac{dy}{dx} = -12x - 3y$$

$$\frac{dy}{dx} = \frac{-12x - 3y}{3x + 4y + 17}$$

Slope at (−1, 0): $\frac{-12(-1) - 3(0)}{3(-1) + 4(0) + 17} = \frac{12}{14} = \frac{6}{7}$

(a) Tangent: $y = \frac{6}{7}(x+1) + 0$ or $y = \frac{6}{7}x + \frac{6}{7}$

(b) Normal: $y = -\frac{7}{6}(x+1) + 0$ or $y = -\frac{7}{6}x - \frac{7}{6}$

23. $2xy + \pi \sin y = 2\pi$

$$2\frac{d}{dx}(xy) + \pi\frac{d}{dx}(\sin y) = \frac{d}{dx}(2\pi)$$

$$2x\frac{dy}{dx} + 2y(1) + \pi\cos y\frac{dy}{dx} = 0$$

$$(2x + \pi\cos y)\frac{dy}{dx} = -2y$$

$$\frac{dy}{dx} = -\frac{2y}{2x + \pi\cos y}$$

Slope at $\left(1, \frac{\pi}{2}\right)$: $-\frac{2(\pi/2)}{2(1) + \pi\cos(\pi/2)} = -\frac{\pi}{2}$

(a) Tangent: $y = -\frac{\pi}{2}(x-1) + \frac{\pi}{2}$ or $y = -\frac{\pi}{2}x + \pi$

(b) Normal: $y = \frac{2}{\pi}(x-1) + \frac{\pi}{2}$ or $y = \frac{2}{\pi}x - \frac{2}{\pi} + \frac{\pi}{2}$

29. $y^2 = x^2 + 2x$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x)$$

$$2yy' = 2x + 2$$

$$y' = \frac{2x+2}{2y} = \frac{x+1}{y}$$

$$y'' = \frac{d}{dx}\left(\frac{x+1}{y}\right)$$

$$= \frac{(y)(1) - (x+1)y'}{y^2}$$

$$= \frac{y - (x+1)\left(\frac{x+1}{y}\right)}{y^2}$$

$$= \frac{y^2 - (x+1)^2}{y^3}$$

31. $\frac{dy}{dx} = \frac{d}{dx}x^{9/4} = \frac{9}{4}x^{(9/4)-1} = \frac{9}{4}x^{5/4}$

33. $\frac{dy}{dx} = \frac{d}{dx}\sqrt[3]{x} = \frac{d}{dx}x^{1/3} = \frac{1}{3}x^{(1/3)-1} = \frac{1}{3}x^{-2/3}$

35. $\frac{dy}{dx} = \frac{d}{dx}(2x+5)^{-1/2} = -\frac{1}{2}(2x+5)^{(-1/2)-1}\frac{d}{dx}(2x+5)$

$$= -\frac{1}{2}(2x+5)^{-3/2}(2) = -(2x+5)^{-3/2}$$

37. $\frac{dy}{dx} = \frac{d}{dx}\left(x\sqrt{x^2+1}\right)$

$$= x\frac{d}{dx}\sqrt{x^2+1} + \sqrt{x^2+1}\frac{d}{dx}(x)$$

$$= x\frac{d}{dx}(x^2+1)^{1/2} + (x^2+1)^{1/2}$$

$$= x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + (x^2+1)^{1/2}$$

$$= x^2(x^2+1)^{-1/2} + (x^2+1)^{1/2}$$

Note: This answer is equivalent to $\frac{2x^2+1}{\sqrt{x^2+1}}$.

39. $\frac{dy}{dx} = \frac{d}{dx}(1 - x^{1/2})^{1/2}$

$$= \frac{1}{2}(1 - x^{1/2})^{-1/2}\frac{d}{dx}(1 - x^{1/2})$$

$$= \frac{1}{2}(1 - x^{1/2})^{-1/2}\left(-\frac{1}{2}x^{-1/2}\right)$$

$$= -\frac{1}{4}(1 - x^{1/2})^{-1/2}x^{-1/2}$$

41. $\frac{dy}{dx}=\frac{d}{dx}3(\csc x)^{3/2}$

$=\frac{9}{2}(\csc x)^{1/2}\frac{d}{dx}(\csc x)$

$=\frac{9}{2}(\csc x)^{1/2}(-\csc x\cot x)$

$=-\frac{9}{2}(\csc x)^{3/2}\cot x$

43. **(a)** If $f(x)=\frac{3}{2}x^{2/3}-3$, then

$f'(x)=x^{-1/3}$ and $f''(x)=-\frac{1}{3}x^{-4/3}$

which contradicts the given equation $f''(x)=x^{-1/3}$.

(b) If $f(x)=\frac{9}{10}x^{5/3}-7$, then

$f'(x)=\frac{3}{2}x^{2/3}$ and $f''(x)=x^{-1/3}$, which matches the given equation.

(c) Differentiating both sides of the given equation

$f''(x)=x^{-1/3}$ gives $f'''(x)=-\frac{1}{3}x^{-4/3}$, so it *must* be true that $f'''(x)=-\frac{1}{3}x^{-4/3}$.

(d) If $f'(x)=\frac{3}{2}x^{2/3}+6$, then $f''(x)=x^{-1/3}$, which matches the given equation.

Conclusion: (b), (c), and (d) could be true.

Section 3.8 Exercises

1. $\frac{dy}{dx}=\frac{d}{dx}\cos^{-1}(x^2)=-\frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}(x^2)$

$=-\frac{1}{\sqrt{1-x^4}}(2x)=-\frac{2x}{\sqrt{1-x^4}}$

5. $\frac{dy}{dt}=\frac{d}{dt}\sin^{-1}\left(\frac{3}{t^2}\right)=\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\frac{d}{dt}\left(\frac{3}{t^2}\right)$

$=\frac{1}{\sqrt{1-\frac{9}{t^4}}}\left(-\frac{6}{t^3}\right)=-\frac{6}{t\sqrt{t^4-9}}$

7. $\frac{dy}{dx}=\frac{d}{dx}(x\sin^{-1}x)+\frac{d}{dx}(\sqrt{1-x^2})$

$=(x)\left(\frac{1}{\sqrt{1-x^2}}\right)+(\sin^{-1}x)(1)+\frac{1}{2\sqrt{1-x^2}}(-2x)$

$=\sin^{-1}x$

9. $x(t)=\sin^{-1}\left(\frac{t}{4}\right)$

$y=\sin^{-1}u \qquad u=\frac{t}{4}$

$\sin y=u \qquad \frac{du}{dt}=\frac{1}{4}$

$\frac{d}{du}(\sin y)=\frac{d}{du}u$

$\cos y\frac{dy}{du}=1$

$\frac{dy}{du}=\frac{1}{\cos y}$

$\frac{d}{du}(\sin^{-1}u)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$v(t)=\frac{1}{4\sqrt{1-t^2/16}}$

$v(3)=\frac{1}{4\sqrt{1-9/16}}=\frac{\sqrt{7}}{7}$