Section 3.8 Exercises

1. $\frac{dy}{dx}=\frac{d}{dx}\cos^{-1}(x^2)=-\frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}(x^2)$

$=-\frac{1}{\sqrt{1-x^4}}(2x)=-\frac{2x}{\sqrt{1-x^4}}$

5. $\frac{dy}{dt}=\frac{d}{dt}\sin^{-1}\left(\frac{3}{t^2}\right)=\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\frac{d}{dt}\left(\frac{3}{t^2}\right)$

$=\frac{1}{\sqrt{1-\frac{9}{t^4}}}\left(-\frac{6}{t^3}\right)=-\frac{6}{t\sqrt{t^4-9}}$

7. $\frac{dy}{dx}=\frac{d}{dx}(x\sin^{-1}x)+\frac{d}{dx}(\sqrt{1-x^2})$

$=(x)\left(\frac{1}{\sqrt{1-x^2}}\right)+(\sin^{-1}x)(1)+\frac{1}{2\sqrt{1-x^2}}(-2x)$

$=\sin^{-1}x$

9. $x(t)=\sin^{-1}\left(\frac{t}{4}\right)$

$y=\sin^{-1}u \qquad u=\frac{t}{4}$

$\sin y=u \qquad \frac{du}{dt}=\frac{1}{4}$

$\frac{d}{du}(\sin y)=\frac{d}{du}u$

$\cos y\frac{dy}{du}=1$

$\frac{dy}{du}=\frac{1}{\cos y}$

$\frac{d}{du}(\sin^{-1}u)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$v(t)=\frac{1}{4\sqrt{1-t^2/16}}$

$v(3)=\frac{1}{4\sqrt{1-9/16}}=\frac{\sqrt{7}}{7}$

13. $\frac{dy}{ds}=\frac{d}{ds}\sec^{-1}(2s+1)$

$=\frac{1}{|2s+1|\sqrt{(2s+1)^2-1}}\frac{d}{ds}(2s+1)$

$=\frac{1}{|2s+1|\sqrt{4s^2+4s}}(2)=\frac{1}{|2s+1|\sqrt{s^2+s}}$

15. $\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}(x^2+1)$

17. $\frac{dy}{dt}=\frac{d}{dt}\sec^{-1}\left(\frac{1}{t}\right)=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\frac{d}{dt}\left(\frac{1}{t}\right)$

$=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\left(-\frac{1}{t^2}\right)=-\frac{1}{\sqrt{1-t^2}}$

Note that the condition $t>0$ is required in the last step

19. $\frac{dy}{dt}=-\frac{d}{dt}\cot^{-1}\sqrt{t-1}=-\frac{1}{1+(\sqrt{t-1})^2}\frac{d}{dt}\sqrt{t-1}$

$=-\left(\frac{1}{1+t-1}\right)\left(\frac{1}{2\sqrt{t-1}}\right)=-\frac{1}{2t\sqrt{t-1}}$

21. $\frac{dy}{dx}=\frac{d}{dx}(\tan^{-1}\sqrt{x^2-1})+\frac{d}{dx}(\csc^{-1}x)$

$=\frac{1}{1+(\sqrt{x^2-1})^2}\frac{d}{dx}(\sqrt{x^2-1})-\frac{1}{|x|\sqrt{x^2-1}}$

$=\frac{1}{x^2}\frac{1}{2\sqrt{x^2-1}}(2x)-\frac{1}{|x|\sqrt{x^2-1}}$

$=\frac{1}{x\sqrt{x^2-1}}-\frac{1}{|x|\sqrt{x^2-1}}$

$=0$

Note that the condition $x>1$ is required in the last step

23.

$y=\sec^{-1}x$

$\frac{dy}{dx}\sec y=\frac{d}{dx}x$

$\sec y\ \tan y\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{1}{\sec y\ \tan y}$

$\frac{dy}{dx}=\frac{1}{x\sqrt{x^2-1}}$

$y'(2)=\frac{1}{2\sqrt{4-1}}=\frac{1}{2\sqrt{3}}=0.289$

$y(2)=\sec^{-1}2=2.203$

$y=0.289(x-2)+2.203$

$y=0.289x+1.625$

27. (a) Since $\frac{dy}{dx}=\sec^2 x$, the slope at $\left(\frac{\pi}{4},1\right)$ is $\sec^2\left(\frac{\pi}{4}\right)=2.$

The tangent line is given by

$y=2\left(x-\frac{\pi}{4}\right)+1$, or $y=2x=\frac{\pi}{2}+1.$

(b) Since $\frac{dy}{dx}=\frac{1}{1+x^2}$, the slope at $\left(1,\frac{\pi}{4}\right)$ is $\frac{1}{1+1^2}=\frac{1}{2}.$

The tangent line is given by $x\neq 0.$

29. **(a)** Note that $f'(x) = -\sin x + 3$, which is always between 2 and 4. Thus f is differentiable at every point on the interval $(-\infty, \infty)$ and $f'(x)$ is never zero on this interval, so f has a differentiable inverse by Theorem 3.

(b) $f(0) = \cos 0 + 3(0) = 1;$

$f'(0) = -\sin 0 + 3 = 3$

(c) Since the graph of $y = f(x)$ includes the point (0, 1) and the slope of the graph is 3 at this point, the graph of $y = f^{-1}(x)$ will include (1, 0) and the slope will be $\frac{1}{3}$,

Thus, $f^{-1}(1) = 0$ and $(f^{-1})'(1) = \frac{1}{3}$.

Section 3.9 Exercises

1. $\frac{dy}{dx} = \frac{d}{dx}(2e^x) = 2e^x$

3. $\frac{dy}{dx} = \frac{d}{dx}e^{-x} = e^{-x}\frac{d}{dx}(-x) = -e^{-x}$

5. $\frac{dy}{dx} = \frac{d}{dx}e^{2x/3} = e^{2x/3}\frac{d}{dx}\left(\frac{2x}{3}\right) = \frac{2}{3}e^{2x/3}$

7. $\frac{dy}{dx} = \frac{d}{dx}(xe^2) - \frac{d}{dx}(e^x) = e^2 - e^x$

11. $\frac{dy}{dx} = \frac{d}{dx}8^x = 8^x \ln 8$

13. $\frac{dy}{dx} = \frac{d}{dx}3^{\csc x} = 3^{\csc x}(\ln 3)\frac{d}{dx}(\csc x)$

$= 3^{\csc x}(\ln 3)(-\csc x \cot x)$

$= -3^{\csc x}(\ln 3)(\csc x \cot x)$

15. $\frac{dy}{dx} = \frac{d}{dx}\ln(x^2) = \frac{1}{x^2}\frac{d}{dx}(x^2) = \frac{1}{x^2}(2x) = \frac{2}{x}$

17. $\frac{dy}{dx} = \frac{d}{dx}\ln(x^{-1}) = \frac{d}{dx}(-\ln x) = -\frac{1}{x}, \ x > 0$

19. $\frac{d}{dx}\ln(\ln x) = \frac{1}{\ln x}\frac{d}{dx}\ln x = \frac{1}{\ln x} \bullet \frac{1}{x} = \frac{1}{x \ln x}$

21. $\frac{dy}{dx} = \frac{d}{dx}(\log_4 x^2) = \frac{d}{dx}\frac{\ln x^2}{\ln 4} = \frac{d}{dx}\left[\left(\frac{2}{\ln 4}\right)(\ln x)\right]$

$= \frac{2}{\ln 4} \bullet \frac{1}{x} = \frac{2}{x \ln 4} = \frac{1}{x \ln 2}$

23. $\frac{dy}{dx} = \frac{d}{dx}\log_2\left(\frac{1}{x}\right) = \frac{d}{dx}(-\log_2 x) = -\frac{1}{x \ln 2}, x > 0$

29. $m = 5$

$y = 3^x + 1$

$y' = 3^x \ln 3 = 5$

$x = 1.379$

$y = 3^{1.379} + 1 = 5.551$

$(1.379, 5.551)$

37. $\frac{d}{dx}\ln(x+2) = \frac{1}{u}\frac{du}{dx}$

$\frac{d}{dx}\ln(u) \qquad u = x + 2$

$f'(x) = \frac{1}{x+2} \qquad \frac{du}{dx} = 1$

$x + 2 > 0$

$x > -2$