Kuta Software - Infinite Calculus 3.1 WKST Name Key

Definition of the Derivative

*Assign odds Date________ Period____

Use the definition of the derivative to find the derivative of each function with respect to x.

1) $y = -2x + 5$

$\lim_{h\to 0} \frac{-2(x+h)+5-(-2x+5)}{h} = \lim_{h\to 0} \frac{-2x-2h+5+2x-5}{h} = \lim_{h\to 0} \frac{-2h}{h} = \boxed{-2}$

2) $f(x) = -4x - 2$

$\lim_{h\to 0} \frac{-4(x+h)-2+4x+2}{h}$

$\lim_{h\to 0} \frac{-4x-4h-2+4x+2}{h} = \lim_{h\to 0} \frac{-4h}{h} = -4$

3) $y = 4x^2 + 1$

$\lim_{h\to 0} \frac{4(x+h)^2+1-4x^2-1}{h} = \lim_{h\to 0} \frac{4x^2+8xh+4h^2-4x^2-1}{h}$

$\lim_{h\to 0} \frac{8xh+4h^2}{h} = \lim_{h\to 0} 8x+4h = \boxed{8x}$

4) $f(x) = -3x^2 + 4$

$\lim_{h\to 0} \frac{-3(x+h)^2+4+3x^2-4}{h} = \lim_{h\to 0} \frac{-3x^2+-6xh-3h^2+4+3x^2-4}{h}$

$\lim_{h\to 0} \frac{-6xh-3h^2}{h} = \lim_{h\to 0} -6x-3h = -6x$

5) $y = -4x^2 - 5x - 2$

$\lim_{h\to 0} \frac{-4(x+h)^2-5(x+h)-2+4x^2+5x+2}{h}$

$\lim_{h\to 0} \frac{-4x^2-8xh-4h^2-5x-5h-2+4x^2+5x+2}{h}$

$\lim_{h\to 0} \frac{-8xh-4h^2-5h}{h} = \lim_{h\to 0} -8x-4h-5 = \boxed{-8x-5}$

6) $y = 3x^2 + 3x + 3$

$\lim_{h\to 0} \frac{3(x+h)^2+3(x+h)+3-3x^2-3x-3}{h}$

$\lim_{h\to 0} \frac{3x^2+6xh+3h^2+3x+3h-3x^2-3x}{h}$

$\lim_{h\to 0} \frac{6xh+3h^2+3h}{h} = \lim_{h\to 0} 6x+3h+3 = \boxed{6x+3}$

7) $y = \sqrt{-3x - 5}$

$\lim_{h\to 0} \frac{\sqrt{-3(x+h)-5}-\sqrt{-3x-5}}{h} \cdot \frac{\sqrt{-3x-3h-5}+\sqrt{-3x-5}}{\sqrt{-3x-3h-5}+\sqrt{-3x-5}}$

$\lim_{h\to 0} \frac{-3x-3h-5+3x+5}{h(\sqrt{-3x-3h-5}+\sqrt{-3x-5})} = \lim_{h\to 0} \frac{-3h}{h(\sqrt{-3x-3h-5}+\sqrt{-3x-5})}$

$\lim_{h\to 0} \frac{-3}{\sqrt{-3x-3h-5}+\sqrt{-3x-5}} = \frac{-3}{2\sqrt{-3x-5}}$

8) $f(x) = \sqrt{4x - 5}$

$\lim_{h\to 0} \frac{\sqrt{4(x+h)-5}-\sqrt{4x-5}}{h} \cdot \frac{\sqrt{4x+4h-5}+\sqrt{4x-5}}{\sqrt{4x+4h-5}+\sqrt{4x-5}}$

$\lim_{h\to 0} \frac{4x+4h-5-4x+5}{h(\sqrt{4x+4h-5}+\sqrt{4x-5})} = \lim_{h\to 0} \frac{4h}{h(\sqrt{4x+4h-5}+\sqrt{4x-5})}$

$\lim_{h\to 0} \frac{4}{(\sqrt{4x+4h-5}+\sqrt{4x-5})} = \frac{4}{2\sqrt{4x-5}} = \frac{2}{\sqrt{4x-5}}$

9) $y = \frac{1}{x + 2}$

$\lim_{h\to 0} \frac{\frac{1}{x+h+2}-\frac{1}{x+2}}{h} = \lim_{h\to 0} \frac{\frac{x+2}{(x+2)(x+h+2)}-\frac{x+h+2}{(x+2)(x+h+2)}}{h}$

$\lim_{h\to 0} \frac{-h}{(x+2)(x+h+2)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2}$

10) $f(x) = -\frac{2}{2x - 1}$

$\lim_{h\to 0} \frac{\frac{-2}{2(x+h)-1}+\frac{2}{2x-1}}{h} = \lim_{h\to 0} \frac{\frac{-2(2x-1)+2(2x+2h-1)}{(2x+2h-1)(2x-1)}}{h}$

$= \lim_{h\to 0} \frac{-4x+2+4x+4h-2}{(2x+2h-1)(2x-1)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(2x+2h-1)(2x-1)}$

$= \frac{4}{(2x-1)(2x-1)} = \frac{4}{4x^2-4x+1}$

Critical thinking question:

11) Use the definition of the derivative to show that $f'(0)$ does not exist where $f(x) = |x|$.

$\lim_{h\to 0} \frac{|x+h|-|x|}{h} = \lim_{h\to 0} \frac{|0+h|-|0|}{h} = \lim_{h\to 0} \frac{|h|}{h}$

$\lim_{h\to 0^+} \frac{|h|}{h} = \frac{.1}{.1} = 1$

$\lim_{h\to 0^-} \frac{|h|}{h} = \frac{|-.1|}{-.1} = \frac{.1}{-.1} = -1$

Not equal!