3.3 Rules for Differentiation Day 2

Power Rule

$4x^{1/2}$

Ex 1) $y = x^4 + 3x^3 - 2x^{-2} + 4\sqrt{x}$

$y' = 4x^3 + 9x^2 + 4x^{-3} + 2x^{-1/2}$

$y' = 4x^3 + 9x^2 + \frac{4}{x^3} + \frac{2}{\sqrt{x}}$

Product Rule

$\frac{d}{dx} f(x)g(x) =$

$\boxed{\frac{d}{dx}(uv) = uv' + vu'}$

$f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$\frac{d}{dx} uv = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

Ex 2) $\frac{d}{dx} \underset{u}{(x - 1)} \underset{v}{(x^2 - 2)}$

$= uv' + vu'$

$= (x-1)(2x) + (x^2-2)1$

$= 2x^2 - 2x + x^2 - 2$

$d/dx = 3x^2 - 2x - 2$

Quotient Rule

$$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}}$$

Ex 3) $\frac{d}{dx}\left(\frac{\overset{u}{x^2 + 2x - 1}}{\underset{v}{x - 4}}\right)$

$$= \frac{vu' - uv'}{v^2} = \frac{(x-4)(2x+2) - (x^2+2x-1)(1)}{(x-4)^2}$$

$$= \frac{\cancel{2x^2} - 8x + \cancel{2x} - 8 - \cancel{x^2} - \cancel{2x} + 1}{(x-4)^2}$$

$$= \frac{x^2 - 8x - 7}{(x-4)^2}$$

Ex 4) $f(x) = x^4 - x^3 + x^2 - 2x + 6$

$f(1) = 1^4 - 1^3 + 1^2 - 2(1) + 6 = \boxed{5}$

Find equations for the tangent and normal lines at x = 1.

$f'(x) = 4x^3 - 3x^2 + 2x - 2$

$f'(1) = 4(1)^3 - 3(1)^2 + 2(1) - 2 = 1 = m$

Tangent $m = 1$ $(1, 5)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 5 = 1(x - 1)}$

Normal $m = -1$ $(1, 5)$

$\boxed{y - 5 = -1(x - 1)}$

Ex 5) $f(x) = x^3 + 3x^2 - 3x + 6$

Value "y"

Where is the tangent line horizontal?

$m = 0$

looking for x coordinate

$$f'(x) = 3x^2 + 6x - 3$$
$$0 = 3x^2 + 6x - 3$$
$$0 = 3(x^2 + 2x - 1)$$

$\sqrt{4}\sqrt{2}$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2}$$
$$= \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

Ex 6) Suppose $u(1) = 2$, $u'(1) = 3$, $v(1) = -2$, $v'(1) = 4$

$$\frac{d}{dx}(uv) = uv' + v \cdot u'$$
$$= 2 \cdot 4 + -2 \cdot 3$$
$$= 8 - 6$$
$$= \boxed{2}$$

$$\frac{d}{dx}\, 2u - 4v + 3uv$$
$$= 2 \cdot \frac{du}{dx} u - 4 \cdot \frac{dv}{dx} v + 3 \cdot \frac{d}{dx} u \cdot v$$
$$= 2 \cdot 3 - 4 \cdot 4 + 3(2 \cdot 4 + -2 \cdot 3)$$
$$= 6 - 16 + 3(2)$$
$$= -4$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v \cdot u' - uv'}{v^2} = \frac{-2(3) - 2 \cdot 4}{(-2)^2}$$
$$= \frac{-6 - 8}{4} = \frac{-14}{4} = \frac{-7}{2}$$

Ex 7) $y = x^4 - x^3 + x^2 - 2x + 6$

$y' = 4x^3 - 3x^2 + 2x - 2$

$y'' = 12x^2 - 6x + 2$

$y''' = 24x - 6$

$y'''' = 24$

$y''''' = 0$

39 $y = 2x^3 - 3x^2 - 12x + 20$

$y' = \boxed{0 = 6x^2 - 6x - 12}$

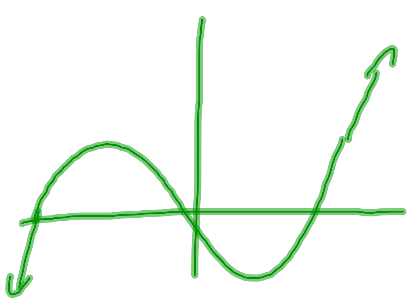
$0 = 6(x^2 - x - 2)$

$0 = 6(x - 2)(x + 1)$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

$x = 2$ $\quad$ $x = -1$

min $\quad$ max



35 $y = x^{-1} + x^2$

$y' = -1x^{-2} + 2x = \frac{-1}{x^2} + 2x$

$y'' = 2x^{-3} + 2 = \frac{2}{x^3} + 2$

$y''' = -6x^{-4} = \frac{-6}{x^4}$

$y'''' = 24x^{-5} = \frac{24}{x^5}$