3.8 Derivatives of Inverse Trigonometric Functions

Day 1

What is an inverse?

- switch x & y around & solve for y
- reflection over $y = x$

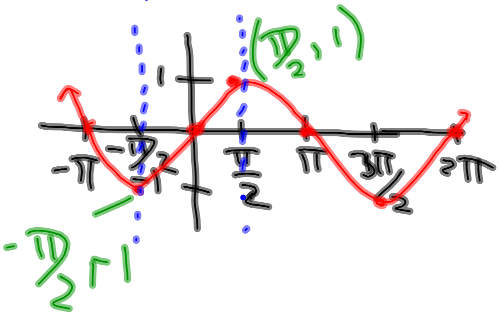
$\sin^{-1}x = \arcsin x$

$\cos^{-1}x = \arccos x$

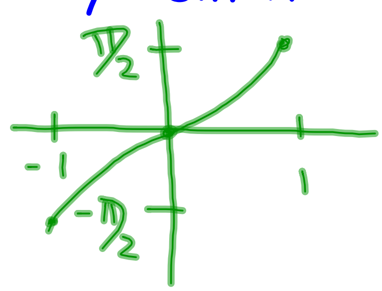
$\tan^{-1}x = \arctan x$

Graphs

$y = \sin x$



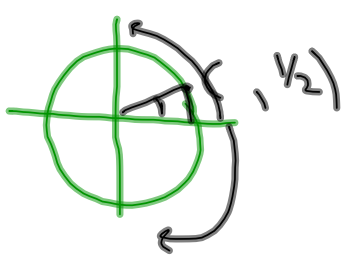
$y = \sin^{-1}x$



$\sin x = 0.5$

$\sin^{-1}(.5) = x$

$x = \frac{\pi}{6}$



MEMORIZE

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

Ex 1) $\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$

$$\frac{d}{dx}\tan^{-1}(x^2) =$$

$u = x^2$

$$= \frac{1}{1+(x^2)^2}\cdot 2x$$

$$= \boxed{\frac{2x}{1+x^4}}$$

Ex 2) $u = \sqrt{3x} = (3x)^{1/2}$

$$\frac{d}{dx}\tan^{-1}\sqrt{(3x)} =$$

$$= \frac{1}{1+(\sqrt{3x})^2}\cdot\frac{1}{2}(3x)^{-1/2}\cdot 3$$

$$= \boxed{\frac{3}{2(1+3x)\sqrt{3x}}}$$

Ex 3)

$$\frac{d}{dx}\sin^{-1}\frac{x}{3} =$$

$\frac{1}{3}x$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\cdot\frac{1}{3}$$

$$= \boxed{\frac{1}{3\sqrt{1-\frac{x^2}{9}}}}$$

Ex 4)

$$\frac{d}{dx}\arcsin(2x^7+1) =$$

$$= \frac{1}{\sqrt{1-(2x^7+1)^2}}\cdot 14x^6$$

$$= \boxed{\frac{14x^6}{\sqrt{1-(2x^7+1)^2}}}$$

Ex 5)

$$\frac{d}{dx}\left(\overset{u\cdot}{x}\,\overset{v}{\cos^{-1}x} + \sqrt{(1-x^2)}\right) =$$

$vu' + uv'$

$(1-x^2)^{1/2}$

$$= \cos^{-1}x\cdot 1 + x\cdot\frac{-1}{\sqrt{1-x^2}} + \frac{1}{\cancel{2}}(1-x^2)^{-1/2}\cdot(-\cancel{2}x)$$

$$= \cos^{-1}x + \frac{-x}{\sqrt{1-x^2}} + \frac{-x}{\sqrt{1-x^2}}$$

$$= \boxed{\cos^{-1}x - \frac{2x}{\sqrt{1-x^2}}}$$