Before we start 4.3, let us take a look at some questions from 4.2.

54. On what interval is the function $g(x) = e^{x^3 - 6x^2 + 8}$ decreasing? $[0,4]$

$$g'(x) = e^{x^3-6x^2+8}(3x^2-12x)$$

① $g'=0$ $\quad 3x^2-12x=0$

$3x(x-4)=0$

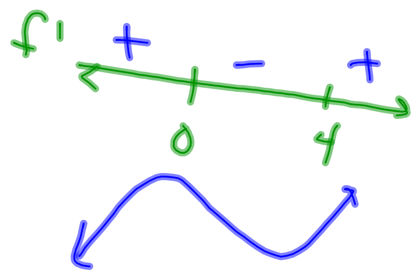
$\boxed{x=0 \quad x=4}$

② $g'=$ und

None

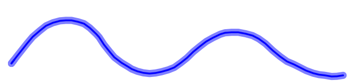
③ Endpts

None

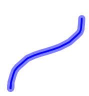


56. All of the following satisfy the conditions of the Mean Value Theorem on the interval [-1,1] except

A) sin x



B) $\sin^{-1} x$



$f' = \frac{1}{\sqrt{1-x^2}}$ und. $x=\pm 1$ still ok

C) $x^{5/3}$

$\sqrt[3]{x^5}$ $\quad f' = \frac{5}{3}x^{2/3} = \frac{5\sqrt[3]{x^2}}{3}$

D) $x^{3/5}$

$\sqrt[5]{x^3}$ $\quad f' = \frac{3}{5}x^{-2/5} = \frac{3}{5\sqrt[5]{x^2}}$ $\quad x=0$ undef

E) $\dfrac{x}{x-2}$

53. If f(x) = cos x, then the Mean Value Theorem guarantees that somewhere between 0 and Π/3, f'(x) =

A) $\frac{-3}{2\Pi}$

B) $-\frac{\sqrt{3}}{2}$

C) $\frac{-1}{2}$

D) 0

E) $\frac{1}{2}$

$f'(x) = -\sin x$

Slope between

$$\frac{f(\pi/3) - f(0)}{\frac{\pi}{3} - 0} = \frac{\frac{1}{2} - 1}{\pi/3}$$

$f(\frac{\pi}{3}) = \cos \pi/3$

$= \frac{1}{2}$

$f(0) = \cos 0 = 1$

$= -\frac{1}{2} \cdot \frac{3}{\pi} = \frac{-3}{2\pi}$

4.3 Connecting f' and f" with the Graph of f

*skip 33 & 37

(2nd Der. Test

First Derivative Test

If f'(x) switches from positive to negative at x=c, then a maximum occurs at x=c.

If f'(x) switches from negative to positive at x=c, then a minimum occurs at x=c.

If f'(x) does not switch signs at x=c, then neither a max nor a min occurs at x=c.

f' $\xleftarrow{\;+\;}\!\!|\!\!\xrightarrow{\;+\;}$ 2

$f'=0$
AT $x=2$

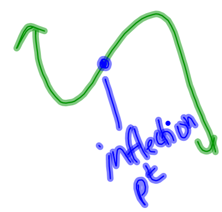
Concavity Test

If $f''(x)>0$ for all x on (a,b), then $f(x)$ is concave up on (a,b) $\cup$ "Concave up like a cup"

If $f''(x)<0$ for all x on (a,b), then $f(x)$ is concave down on (a,b) $\cap$ "Concave down like a frown"

Inflection Points

If $f''(x)$ switches signs at $x=c$, then $x=c$ is an inflection point.



So we will look at $f''=0$ and $f''=$ und to find inflection points.

Ex 1) Use $f'(x)$ and $f''(x)$ to determine increasing/decreasing, max/min, concavity, and inflection points. $\boxed{f(x) = x^3 - 12x - 5}$

$f'(x)=3x^2-12$

① $f'=0$ $\quad 0=3x^2-12$

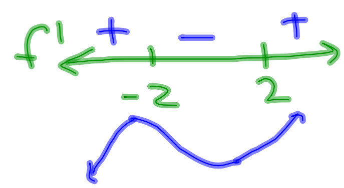
$12=3x^2$

$4=x^2$

$\boxed{x=\pm 2}$

② $f'=$ und $\boxed{\text{None}}$

③ Endpts None



Local Max: $x=-2$, $y=11$

Local Min: $x=2$, $y=-21$

incr: $(-\infty,-2]$ $[2,\infty)$

decr: $[-2,2]$

Ex1) Continued.... $f(x) = x^3 - 12x - 5$

$f' = 3x^2 - 12$

$f'' = 6x$

① $f'' = 0$ $\quad 6x = 0$

$x = 0$

③ $f'' =$ und.

never

f''

(−) (+)

$x = 0$

$f'' > 0$, Concave Up

$f'' < 0$, Concave down

$x = 0$, inflection point

Ex 2) Increasing: [a, c] [e, f]

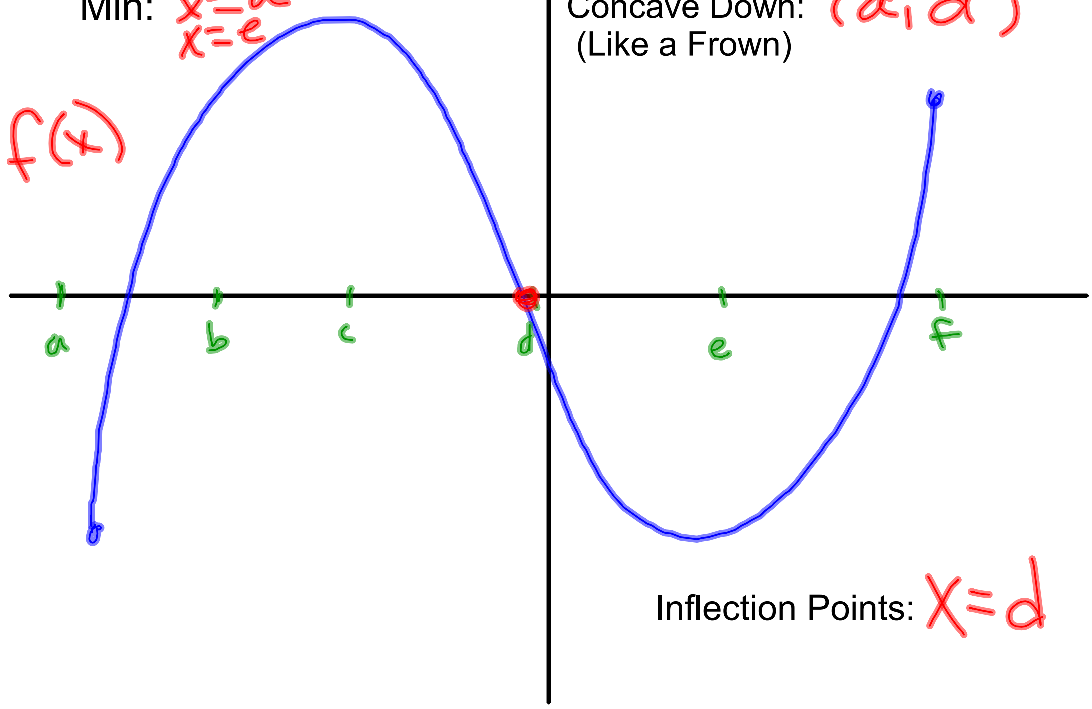
Decreasing: [c, e]

Max: $x = c$, $x = f$

Min: $x = a$, $x = e$

Concave Up: (d, f)
(Like a Cup)

Concave Down: (a, d)
(Like a Frown)



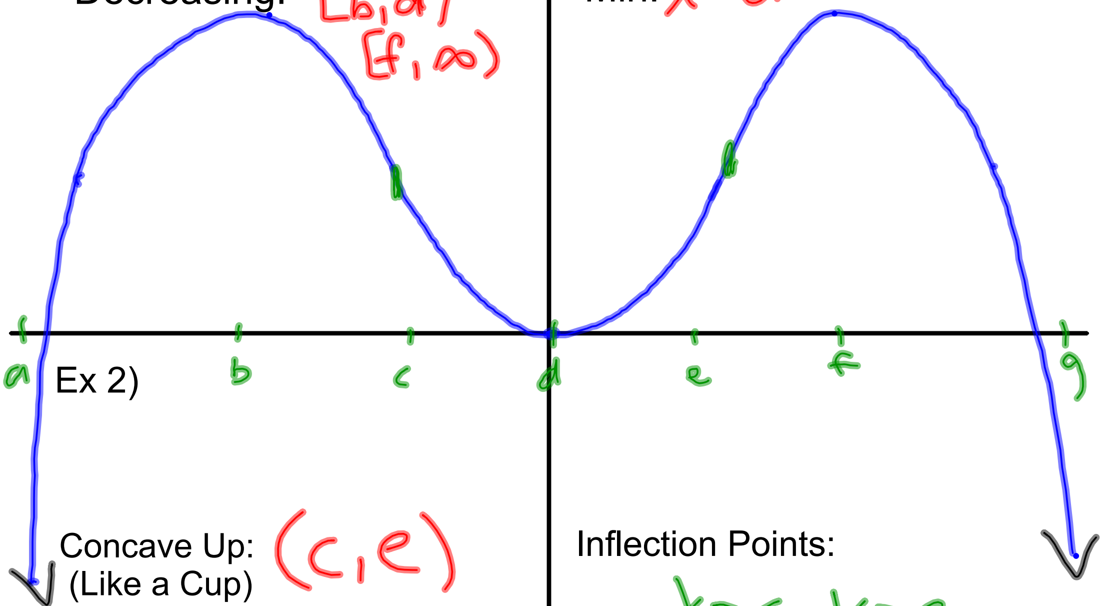
Inflection Points: $x = d$

Ex 2) Increasing: $(-\infty, b]$
$[d, f]$

Decreasing: $[b, d]$
$[f, \infty)$

Max: $x = b,\ x = f$

Min: $x = d$



Concave Up: (c, e)
(Like a Cup)

Concave Down: $(-\infty, c)$
(e, ∞)
(Like a Frown)

Inflection Points:

$x = c,\ x = e$