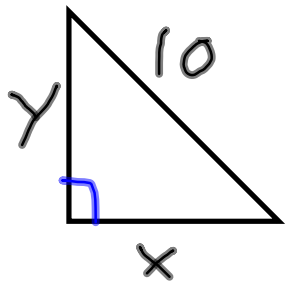
Questions from 4-4...

$vh' + uv'$

55. What is the maximum area of a right triangle with hypotenuse 10?



$A(x) = \frac{1}{2} b \cdot h$

$A(x) = \frac{1}{2} \cdot x \cdot \sqrt{100-x^2}$

$u \cdot v$

$y^2 + x^2 = 100$

$y^2 = 100 - x^2$

$y = \pm\sqrt{100-x^2}$

$A'(x) = \sqrt{100-x^2} \cdot \frac{1}{2} + \frac{1}{2}x \cdot \frac{1}{2}(100-x^2)^{-\frac{1}{2}} \cdot -2x$

$= \frac{\sqrt{100-x^2}}{2} + \frac{-x^2}{2\sqrt{100-x^2}}$

$\frac{\sqrt{100-x^2}}{1} \cdot \frac{2\sqrt{100-x^2}}{2\sqrt{100-x^2}}$

$= \frac{2(50-x^2)}{2\sqrt{100-x^2}}$

$= \frac{100-x^2-x^2}{2\sqrt{100-x^2}} = \frac{100-2x^2}{2\sqrt{100-x^2}}$

$= \frac{50-x^2}{\sqrt{100-x^2}}$

$50 - x^2 = 0$

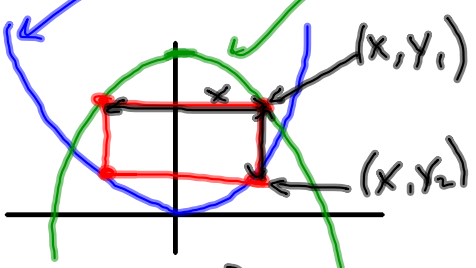
$x^2 = 50$

$x = \pm\sqrt{50}$

$A(\sqrt{50}) = \frac{1}{2}\sqrt{50} \cdot \sqrt{100-(\sqrt{50})^2}$

$= \frac{1}{2} \cdot \sqrt{50} \cdot \sqrt{50} = \frac{1}{2} \cdot 50 = 25 \text{ units}^2$

56. A rectangle is inscribed between the parabolas $y=4x^2$ and $y=30-x^2$, what is the maximum area of such a rectangle?



$A(x) = l \cdot w$

$A(x) = 2x \cdot (30 - x^2 - 4x^2)$

$A(x) = 2x(30 - 5x^2)$

$A(x) = 60x - 10x^3$

$A'(x) = 60 - 30x^2$

$0 = 60 - 30x^2$

$30x^2 = 60$

$x^2 = 2$

$x = \pm\sqrt{2}$

$\sqrt{2}^3 = \sqrt{2}^2 \cdot \sqrt{2}$

$A(\sqrt{2}) = 60\sqrt{2} - 10\sqrt{2}^3$

$= 60\sqrt{2} - 20\sqrt{2}$

$= 40\sqrt{2}$ units^2

#25 Thm 7 P.224

Minimizing Avg Cost

Avg Cost = Marginal Cost

C'

$\frac{C(x)}{x}$ = avg. cost of producing x-items