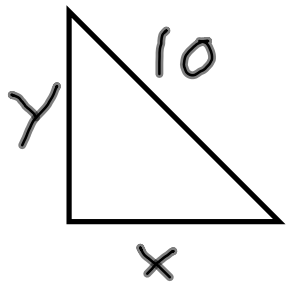
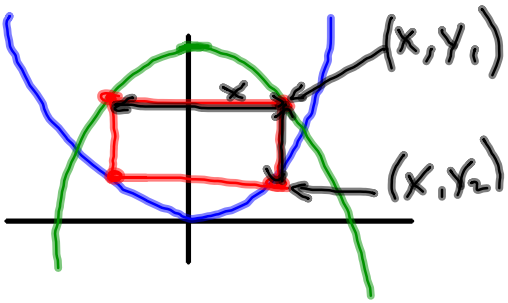
Questions from 4-4...

55. What is the maximum area of a right triangle with hypotenuse 10?



56. A rectangle is inscribed between the parabolas $y=4x^2$ and $y=30-x^2$, what is the maximum area of such a rectangle?



4.5 Linearization and Newton's Method

tangent line

Ex 1) Find the linearization of $f(x) = x^2+4x+1$ at $x = 2$. Use the linearization to approximate $f(2.1)$.

① Tangent Line: $x=2$ $\quad 2^2+4(2)+1$ $\quad 4+8+1$ $\quad 13$

$f'(x) = 2x+4$

$f'(2) = 2\cdot 2+4 = \boxed{8=m}$ $\quad (2,13)$

$y-13 = 8(x-2)$

$\boxed{y = 8x-3}$

② $y(2.1) = 8(2.1)-3 = \boxed{13.8}$ Approximate

Actual

$f(2.1) = (2.1)^2+4(2.1)+1 = 13.81$

Ex 2) Find the linearization of f(x) = ln (x +1) at x = 0. Use the linearization to approximate f(0.1).

① Tangent line:

$f'(x) = \frac{1}{x+1} \cdot 1$

$f'(0) = \frac{1}{0+1} \cdot 1 = 1 = m$

$\ln(0+1)$

$\ln 1 = 0$

$(0, 0)$

$y - 0 = 1(x - 0)$

$y = x$

② Approximate: $y(.1) = .1$

Ex 3) Find the linearization of $f(x) = e^x + \sin x$ at $x=0$. Use the linearization to approximate $f(0.1)$.

① Tangent: $f'(x) = e^x + \cos x$

$f'(0) = e^0 + \cos 0$

$= 1 + 1 = 2 = m$

$(0, 1)$

$e^0 + \sin 0$

$1 + 0 = 1$

$y - 1 = 2(x - 0)$

$y = 2x + 1$

② Approx: $y(.1) = 2(.1) + 1$

$= 1.2$

Ex 4) $y = x^3+2x^2+ x - 5$

Assume x = 1 and dx = 0.05, find dy.

$$y' = \frac{dy}{dx} = 3x^2 + 4x + 1$$

$$\frac{dy}{.05} = 3(1)^2 + 4(1) + 1$$

$$\frac{dy}{.05} = 8$$

$$dy = 8(.05)$$

$$\boxed{dy = .4}$$

Ex 5) $y = \dfrac{2x^{a}}{\underbrace{1+x^2}_{v}}$

Assume x = -2 and dx = 0.1, find dy.

$$\frac{dy}{dx} = \frac{(1+x^2)\cdot 2 - 2x(2x)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{2+2x^2-4x^2}{(1+x^2)^2} \qquad (-2x^2)$$

$$\frac{dy}{.1} = \frac{2+-2(-2)^2}{(1+(-2)^2)^2}$$

$$(.1)\frac{dy}{.1} = \frac{-6}{25}(.1)$$

$$dy = -.024$$

Ex 6) $2y = x^2 - xy$ (u·v) $vu' + uv'$

Assume x = 2 and dx = -0.05, find dy.

$2y = 4 - 2y$

$4y = 4$

$y = 1$

$$2 \cdot \frac{dy}{dx} = 2x - (y \cdot 1 + x \cdot \frac{dy}{dx})$$

$$2\frac{dy}{dx} = 2x - y - x\frac{dy}{dx}$$

$$2\frac{dy}{dx} + x\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(2 + x) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{2 + x}$$

$$\frac{dy}{-.05} = \frac{2(2) - 1}{2 + 2}$$

$$(-.05)\frac{dy}{-.05} = \frac{3}{4}(-.05)$$

$dy = -\frac{3}{80} = -.0375$