5.3 Definite Integrals and Antiderivatives

Rules for Definite Integrals

A specific starting and stopping point

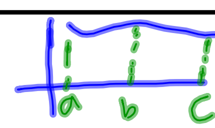
$$\int_a^b f(x)\,dx$$

Zero Property

$$\int_a^a f(x)\,dx = 0$$

Constant Multiple Rule

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$



$$\int_b^c f(x)\,dx + \int_a^b f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Ex 1) **Given**

$$\int_1^9 f(x)\,dx = -1$$

$$\int_7^9 f(x)\,dx = 5$$

$$\int_7^9 h(x)\,dx = 4$$

$$\int_7^9 (f(x) + h(x))\,dx = 5 + 4 = 9$$

$$\int_9^1 f(x)\,dx = -\int_1^9 f(x)\,dx = -(-1) = 1$$

$$\int_7^9 3h(x)\,dx = 3\int_7^9 (h(x))\,dx = 3 \cdot 4 = 12$$

$$\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx = -1 - 5 = -6$$

$$\int_9^7 (h(x) - f(x))\,dx = -4 - -5 = 1$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

F(x) is the anti-derivative of f(x)

f(x) is the derivative of F(x)

$f(x) = x^3$

$f'(x) = 3x^2$

$\boxed{\text{Note:}}$ $\boxed{\int x^r \cdot dr = \frac{x^{r+1}}{r+1} + C}$

$\int (3x^2 + 4)\,dx = \frac{3x^{2+1}}{3} + 4x + C$

$= x^3 + 4x + C$

$f(x) = 3x^2 + 7x + 8$

$f'(x) = 6x + 7$

Ex 2) f(x) = 2x F(x) = $\frac{2x^{1+1}}{2} = \boxed{x^2}$

f(x) = cos x F(x) = $\sin x$

f(x) = -sin x F(x) = $\cos x$

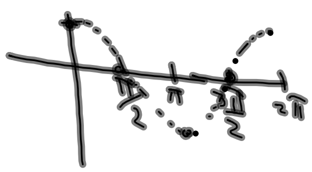
f(x) = e^x F(x) = e^x

f(x) = 1/x F(x) = $\ln x$

f(x) = 4 F(x) = $4x$

Ex 3) $\int_1^7 4\,dx = 4x\Big|_1^7 = 4\cdot 7 - 4\cdot 1 =$
$= 28 - 4 = 24$

Ex 4) $\int_0^{2\pi} \cos x\,dx = \sin x\Big|_0^{2\pi} = \sin(2\pi) - \sin 0$
$= 0 - 0 = 0$



Ex 5) $\int_1^4 2x\,dx = \frac{2x^{1+1}}{2} = x^2\Big|_1^4 = 4^2 - 1^2$
$= 16 - 1 = 15$

Ex 6) $\int_{-2}^5 3x^2\,dx = \frac{3x^{2+1}}{3} = x^3\Big|_{-2}^5 = 5^3 - (-2)^3$
$= 125 + 8$
$= 133$

Average Value (y-value)

$$= \frac{1}{b-a}\left(\int_a^b f(x)\,dx\right)$$

* Need to Memorize formula... Will be on AP Test.

Ex 7) Find the average value of $f(x) = 3x^2 - 1$ on $[0, 4]$.

a b

$$\text{Average Value} = \frac{1}{4-0}\int_0^4 (3x^2-1)\,dx$$

$$= \frac{1}{4}\left(\frac{3x^{2+1}}{3} - x\right)\Big|_0^4$$

$$= \frac{1}{4}\left(x^3 - x\right)\Big|_0^4$$

$$= \frac{1}{4}\left(4^3 - 4 - (0-0)\right)$$

$$= \frac{1}{4}(60) = 15$$ → Average y value of f(x) from [0,4]