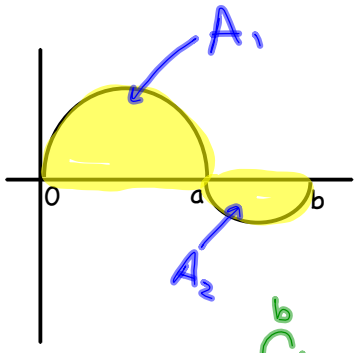
5.4 day 3

Total Area

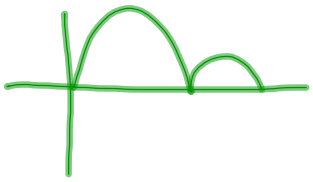


$$\int_0^b f(x)\,dx = A_1 - A_2$$

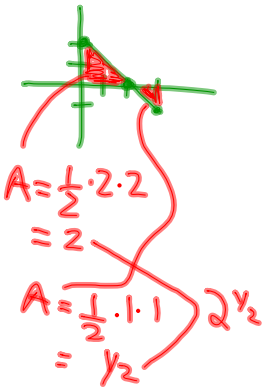
$$\int_0^a f(x)\,dx + \left|\int_a^b f(x)\,dx\right|$$

Abs Value

$$\int_0^b |f(x)|\,dx$$



Ex 1) Find the total area between f(x) and the x-axis.
f(x) = 2 − x [0, 3]



$$\int_0^2 (2-x)\,dx + \left|\int_2^3 (2-x)\,dx\right|$$

$$\int_0^3 |2-x|\,dx$$

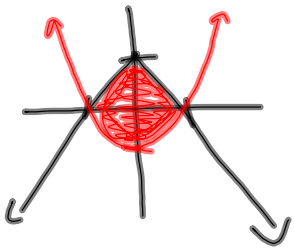
$$2x - \frac{x^2}{2}\Big|_0^2 + \left|\left(2x - \frac{x^2}{2}\right)\Big|_2^3\right|$$

$$\left((4-2)-0\right) + \left|\left(6-\frac{9}{2}\right)-(4-2)\right|$$

$$2 + \tfrac{1}{2} \qquad \left|\tfrac{3}{2}-2\right| = \left|-\tfrac{1}{2}\right|$$

$2\frac{1}{2}$

Ex 2) Find the area enclosed between the f(x) = 1 −|x| and f(x) = x^2 − 1.



$$\int_{-1}^{1}(1-|x|)\,dx + \left|\int_{-1}^{1}(x^2-1)\,dx\right|$$

$$\frac{1}{2}\cdot 2\cdot 1 + \left|\left(\frac{x^3}{3}-x\right)\Big|_{-1}^{1}\right|$$

$$1 + \left|\left(\frac{1}{3}-1\right)-\left(-\frac{1}{3}+1\right)\right|$$

$$1 + \left|-\frac{2}{3}-\frac{2}{3}\right|$$

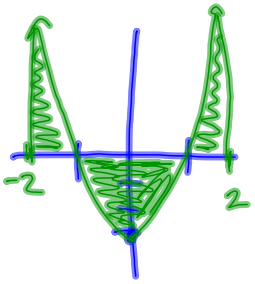
$$1 + \left|-\frac{4}{3}\right|$$

$$1 + 4/3 = \boxed{\frac{7}{3}}\ u^2$$

Ex 4) Find the total area of the region between the curve and the x-axis if f(x) = $3x^2$ − 3, [−2, 2].

$$2\int_{1}^{2}(3x^2-3)\,dx + \left|\int_{-1}^{1}(3x^2-3)\,dx\right|$$

$$3x^2-3=0$$
$$3x^2=3$$
$$x^2=1$$
$$x=\pm 1$$



$$2\left((x^3-3x)\Big|_{1}^{2}\right) + \left|(x^3-3x)_{-1}^{1}\right|$$

$$2\left((8-6)-(1-3)\right) + \left|(1-3)-(-1+3)\right|$$

$$2(2+2) \quad + \quad |-2-2|$$

$$8 \quad + \quad 4$$

$$\boxed{12\ u^2}$$