6.1 Slope Fields and Euler's Method

Find the general solution to the exact differential equation.

Ex 1) $\frac{dy}{dx} = 2x$ $\quad \int 2x \cdot dx$ $\quad y = x^2 + C$

$\int x^r = \frac{x^{r+1}}{r+1} + C$ $\quad \frac{2x^{1+1}}{2} = x^2$

Ex 2) $\frac{dy}{dx} = \sec(x)\tan(x) - e^x$ $\quad y = \sec x - e^x + C$

$\int (\sec x \tan x - e^x)dx$

Ex 3) $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}$ $\quad y = \ln x - \frac{-1}{x} + C = \ln x + \frac{1}{x} + C$

$\int \left(\frac{1}{x} - x^{-2}\right)dx$

$\frac{x^{-2+1}}{-1} = \frac{x^{-1}}{-1} = \frac{-1}{x}$

Find the general solution to the exact differential equation.

Ex 4) $\frac{dy}{dx} = 2x(\cos x^2)$ $\quad y = \sin x^2 + C$

$\int 2x(\cos x^2)$

Check: $y = \sin x^2$, $y' = \cos x^2 (2x)$

Ex 5) $\frac{dy}{dx} = \sec^2 x + 2x + 5$ $\quad y = \tan x + x^2 + 5x + C$

$\int (\sec^2 x + 2x + 5)dx$ $\quad \frac{2x^2}{2}$

Ex 6) $\frac{dy}{dx} = e^x - 6x^2$ $\quad y = e^x - 2x^3 + C$

$\int (e^x - 6x^2)$

Solve the initial value problem.

Ex 7) $\frac{dy}{dx} = 2\cos(x)$ $\int 2\cos x$

"f" Graph goes through this point. → $y = 3, x = 0$

$y = 2\sin x + C$

$3 = 2\sin 0 + C$

$3 = 0 + C$

$3 = C$

$y = 2\sin x + 3$

Ex 8) $\frac{dy}{dx} = 2e^x - \cos x$

$y = 3, x = 0$

$\int (2e^x - \cos x)$

$2\int e^x - \int \cos x$

$y = 2e^x - \sin x + C$

$3 = 2e^0 - \sin 0 + C$

$3 = 2 - 0 + C$

$1 = C$

$y = 2e^x - \sin x + 1$

Solve the initial value problem.

Ex 9)

$\frac{dy}{dx} = 10x^9 + 5x^4 - 2x + 4$

$f(1) = 6$

$x = 1$

$y = 6$

$y = x^{10} + x^5 - x^2 + 4x + C$

$6 = 1 + 1 - 1 + 4 + C$

$6 = 5 + C$

$1 = C$

$y = x^{10} + x^5 - x^2 + 4x + 1$

Ex 10)

$\frac{dy}{dx} = \frac{1}{x^2} - \frac{2}{x^3}$

$f(1) = 4$

$x = 1$

$y = 4$

$\int (x^{-2} - 2x^{-3})dx$

$\frac{x^{-1}}{-1} - \frac{2x^{-2}}{-2} = \frac{-1}{x} + \frac{1}{x^2} + C$

$4 = \frac{-1}{1} + \frac{1}{1^2} + C$

$4 = C$

$y = \frac{-1}{x} + \frac{1}{x^2} + C$

$y = \frac{-1}{x} + \frac{1}{x^2} + 4$