6.2 Antidifferentiation by Substitution

Definite Integral $\int_a^b f(x)\,dx$ tomorrow

Indefinite Integral $\int f(x)\,dx$ today

U - Substitution

Opposite of the chain rule

(Used when one function is inside of another)

Ex 1) $\int 3t^2(\cos t^3)\,dt$

let $u = t^3$

$\frac{du}{dt} = 3t^2$

$du = 3t^2 \cdot dt$

$\int \cos(u) \cdot du$

$= \sin u + C = \boxed{\sin t^3 + C} = y$

Ex 2) $\int (\sin 3x)\, dx$

let $u = 3x$

$\frac{du}{dx} = 3$

$\frac{1}{3} du = \cancel{\frac{3}{3}} dx$

$\int \sin u \cdot \frac{1}{3} \cdot du$

$\frac{1}{3} \int \sin u \cdot du$

$\frac{1}{3} \cdot -\cos u + C = \boxed{-\frac{1}{3}\cos(3x) + C}$

Check $-\frac{1}{3} \cdot -\sin(3x) \cdot 3$

Ex 3) $\int (28)(7x - 2)^3\, dx$

let $u = 7x - 2$

$\frac{du}{dx} = 7$

$du = 7 \cdot dx$

$\frac{1}{7} \cdot du = dx$

$28 \int (7x-2)^3 dx$

$28 \cdot \int u^3 \cdot \frac{1}{7} du$

$28 \cdot \frac{1}{7} \int u^3 \cdot du$

$4 \cdot \frac{u^4}{4} + C = u^4 + C = \boxed{(7x-2)^4 + C}$

Recall

$\int x^r = \frac{x^{r+1}}{r+1}$

Check

$4(7x-2)^3 \cdot 7$

Ex 4) $\int \sqrt{\cot x}\, \csc^2 x\, dx$

let

$u = \cot x$

$\frac{du}{dx} = -\csc^2 x$

$du = -\csc^2 x\, dx$

$-du = \csc^2 x \cdot dx$

$-\int \sqrt{u} \cdot du$

$-\int u^{1/2} \cdot du$

$-\frac{u^{3/2}}{\frac{3}{2}} = -\frac{2}{3} u^{3/2} + C = -\frac{2}{3}(\cot x)^{3/2} + C$

Ex 5) $\int (x)(\cos 2x^2)\, dx$

let

$u = 2x^2$

$\frac{du}{dx} = 4x$

$du = 4x \cdot dx$

$\frac{1}{4} \cdot du = x \cdot dx$

$\int \cos u \cdot \frac{1}{4} \cdot du$

$\frac{1}{4} \int \cos u \cdot du$

$\frac{1}{4} \sin u + C = \frac{1}{4} \sin(2x^2) + C$

Ex 6) $\int \frac{\ln^6 x}{x}\,dx$

$(\ln x)^6$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} \cdot dx$

$\int u^6 \cdot du$

$\frac{u^7}{7} + C = \boxed{\frac{(\ln x)^7}{7} + C}$

Check $\frac{\cancel{7}(\ln x)^6}{\cancel{7}} \cdot \frac{1}{x}$

Ex 7) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$

let $u = \cos(2t+1)$

$\frac{du}{dt} = -\sin(2t+1) \cdot 2$

$du = -2\sin(2t+1) \cdot dt$

$-\frac{1}{2}du = \sin(2t+1) \cdot dt$

$\int \frac{1}{u^2} \cdot \frac{-1}{2} du$

$-\frac{1}{2}\int \frac{1}{u^2} \cdot du$

$-\frac{1}{2}\frac{u^{-2+1}}{-1} + C = \frac{1}{2} \cdot \frac{1}{u} + C = \frac{1}{2} \cdot \frac{1}{\cos(2t+1)} + C$

$= \boxed{\frac{1}{2} \cdot \sec(2t+1) + C}$

Ex 8) $\int (\sec^2 x)\, dx = \boxed{\tan x + C}$